

# New Formulas and Predictions for Running Fermion Masses at Higher Scales in SM, 2HDM, and MSSM

C.R. Das\* and M.K. Parida †

*Physics Department, North-Eastern Hill University*

*Shillong 793022, India*

(October 1, 2000)

## Abstract

Including contributions of scale-dependent vacuum expectation values, we derive new analytic formulas and obtain substantially different numerical predictions for the running masses of quarks and charged-leptons at higher scales in the SM, 2HDM and MSSM. These formulas exhibit significantly different behaviours with respect to their dependence on gauge and Yukawa couplings than those derived earlier. At one-loop level the masses of the first two generations are found to be independent of Yukawa couplings of the third generation in all the three effective theories in the small mixing limit. Analytic formulas are also obtained for running  $\tan \beta(\mu)$  in 2HDM and MSSM. Other numerical analyses include study of the third generation masses at high scales as functions of low-energy values of  $\tan \beta$  and SUSY scale  $M_S = M_Z - 10^4$  GeV.

Typeset using REVTEX

---

\*crdas@email.com

†mparida@dte.vsnl.net.in

## I. INTRODUCTION

One of the most attractive features of current investigations in gauge theories is the remarkable unification of the gauge couplings of the standard model (SM) at the SUSY GUT scale,  $M_U = 2 \times 10^{16}$  GeV, when extrapolated through the minimal supersymmetric standard model (MSSM) [1]. Although the nonsupersymmetric standard model (SM), or the two-Higgs doublet model (2HDM) do not answer the question of gauge hierarchy, unification of the gauge couplings is also possible at the corresponding GUT scales when they are embedded in nonSUSY theories like  $SO(10)$  and the symmetry breaking takes place in two steps with left-right models as intermediate gauge symmetries [2]. Grand unification of gauge couplings of the SM in single-step breakings of GUTs has also been observed when the grand desert contains additional scalar degrees of freedom [3] and the minimal example is a  $\xi(3,0,8)$  of SM contained in  $\underline{75} \subset SU(5)$  or  $\underline{210} \subset SO(10)$  with mass  $M_\xi = 10^{11} - 10^{13}$  GeV [4]. Unification of gauge couplings in nonSUSY  $SO(10)$  has been demonstrated with relatively large GUT-threshold effects [5]. Yukawa coupling unification at the intermediate scale has also been observed in nonSUSY  $SO(10)$  with 2HDM as the weak scale effective gauge theory [6]. Apart from unity of forces at high scales, SM, 2HDM and MSSM have tremendous current importance as effective theories as they emerge from a large class of fundamental theories.

Recent experimental evidences in favour of neutrino masses and mixings have triggered an outburst of models many of which require running masses and mixings of quarks and charged-leptons at high scales as inputs for obtaining predictions in the neutrino sector [7]. The running masses are not only essential at the weak scale, but they are also required at the intermediate and the GUT scales in order to testify theories based upon quark-lepton unification with different Yukawa textures and for providing unified explanation of all fermion masses [8–12]. Quite recently extrapolation of running masses and couplings to high scales have been emphasized as an essential requirement for testing more fundamental theories [12].

In a recent paper one of us (M.K.P) and Purkayastha [13] have obtained new analytic formulas and numerical estimations for the fermion masses at higher scales in MSSM including contributions of scale-dependent vacuum expectation values (VEVs) where the SUSY scale ( $M_S$ ) was assumed to be close to the weak scale ( $M_S \approx M_Z$ ). In this paper we extend such investigations to SM, 2HDM and MSSM with the SUSY scale  $M_S \geq O$  (TeV).

It is also possible that in a different renormalisation scheme, similar to that formulated by Sirlin et. al. [14], the VEVs themselves do not run when they are expressed in terms of physical parameters defined on the mass shell. This makes it possible to avoid separate running of the VEVs and Yukawa couplings, but have just the fermion masses directly as running quantities. While it would be quite interesting to examine the consequences of such a scheme, the purpose of the present and the recent works [13] is to address the outcome of the most frequently exploited renormalisation scheme where the Yukawa couplings and the VEVs run separately [15–23].

This paper is organised in the following manner. In Sec. II we cite examples where running VEVs have been exploited by a number of authors and state relevant renormalisation group equations (RGEs). In Sec. III we derive analytic formulas. In Sec. IV we show how the formulas derived earlier for MSSM are modified when  $M_S \gg M_Z$ . Numerical predictions at higher scales are reported in Sec. V. Summary and conclusions are stated in Sec. VI.

## II. RGES FOR COUPLINGS AND VACUUM EXPECTATION VALUES

After the pioneering discovery of  $b - \tau$  unification at the nonSUSY  $SU(5)$  GUT scale [24], a number of theoretical investigations have been made to examine the behaviour of Yukawa couplings and running masses at higher scales. Following the frequently exploited renormalisation scheme [15–23] where the Yukawa couplings and the VEVs run separately, the running Dirac mass of the fermion ‘ $a$ ’ is defined as,

$$M_a(\mu) = Y_a(\mu)v_a(\mu) \quad (2.1)$$

Then the running of  $M_a(\mu)$  is governed by the RGEs of  $Y_a(\mu)$  and  $v_a(\mu)$  both. To cite some examples, Grimus [21] has derived approximate analytic formulas in SM for all values of  $\mu$  extending upto the nonSUSY  $SU(5)$  GUT scale utilising the corresponding scale-dependent VEV. In the discovery of fixed point of Yukawa couplings, Pendleton and Ross [22] have exploited the RGE of the SM Higgs VEV to derive the RGEs of the running masses from  $\mu = M_W - M_{\text{GUT}}$ . Anomalous dimensions occurring in the RGEs of respective VEVs have been explicitly derived and stated up to two-loops by Arason et. al. [15,16] and by Castano, Pirad and Ramond [17] for SM and MSSM. While investigating renormalisation of the neutrino mass operator, Babu, Leung and Pantaleone [23] have derived RGE for  $\tan \beta(\mu)$  in a class of 2HDM as a consequence of running VEVs in the model. More recently Balzeleit et. al. [19] have utilised the RGE of the VEV in SM to determine running masses for  $\mu = M_W - 10^{10}$  GeV. Cvetic, Hwang and Kim [20] have derived RGEs for the VEVs in 2HDM and utilised them to obtain running quark-lepton masses at high scales and also investigate suppression of flavour changing neutral current in the model. Most recently the RGEs of running VEVs have been utilised by one of us (MKP) and Purkayastha [13] who have obtained new analytic formulas and numerical estimations of the fermion masses at higher scales taking the SUSY scale  $M_S \approx M_Z$ .

We consider only the class of 2HDM where  $\Phi_u$  gives masses to up-quarks and  $\Phi_d$  to down-quarks and charged-leptons. For the sake of simplicity we ignore neutrino mass in the present paper which will be addressed separately. Our definitions and conventions for the Yukawa couplings and masses are governed by the following Yukawa Lagrangian (Superpotential) in SM or 2HDM (MSSM) and the corresponding VEVs of Higgs scalars,

SM

$$\mathcal{L}_Y = \overline{Q}_L Y_U \tilde{\Phi} U_R + \overline{Q}_L Y_D \Phi D_R + \overline{l}_L Y_E \Phi E_R + h.c.$$

$$\langle \Phi^0(\mu) \rangle = v(\mu) \tag{2.2}$$

2HDM, MSSM

$$\mathcal{L}_Y = \overline{Q}_L Y_U \Phi_u U_R + \overline{Q}_L Y_D \Phi_d D_R + \overline{l}_L Y_E \Phi_d E_R + h.c.$$

$$\langle \Phi_u^0(\mu) \rangle = v_u(\mu) = v(\mu) \sin \beta(\mu)$$

$$\langle \Phi_d^0(\mu) \rangle = v_d(\mu) = v(\mu) \cos \beta(\mu)$$

$$v^2(\mu) = v_u^2(\mu) + v_d^2(\mu)$$

$$\tan \beta(\mu) = v_u(\mu)/v_d(\mu) \quad (2.3)$$

The relevant RGEs for the Yukawa matrices at one-loop level for the three effective theories are expressed as [15–18,25–27],

$$\begin{aligned} 16\pi^2 \frac{dY_U}{dt} &= \left[ \text{Tr} \left( 3Y_U Y_U^\dagger + 3aY_D Y_D^\dagger + aY_E Y_E^\dagger \right) + \frac{3}{2} \left( bY_U Y_U^\dagger + cY_D Y_D^\dagger \right) - \sum_i C_i^{(u)} g_i^2 \right] Y_U \\ 16\pi^2 \frac{dY_D}{dt} &= \left[ \text{Tr} \left( 3aY_U Y_U^\dagger + 3Y_D Y_D^\dagger + Y_E Y_E^\dagger \right) + \frac{3}{2} \left( bY_D Y_D^\dagger + cY_U Y_U^\dagger \right) - \sum_i C_i^{(d)} g_i^2 \right] Y_D \\ 16\pi^2 \frac{dY_E}{dt} &= \left[ \text{Tr} \left( 3aY_U Y_U^\dagger + 3Y_D Y_D^\dagger + Y_E Y_E^\dagger \right) + \frac{3}{2} bY_E Y_E^\dagger - \sum_i C_i^{(e)} g_i^2 \right] Y_E \end{aligned} \quad (2.4)$$

The RGEs for the VEV in the SM has been derived up to two-loop from wave function renormalisation of the scalar field [15,16,18,19,21,22] and the one-loop equation is,

$$16\pi^2 \frac{dv}{dt} = \left[ \sum_i C_i^v g_i^2 - \text{Tr} \left( 3Y_U Y_U^\dagger + 3Y_D Y_D^\dagger + Y_E Y_E^\dagger \right) \right] v \quad (2.5)$$

where  $t = \ln \mu$ .

The RGEs for  $v_a$  ( $a = u, d$ ) in the 2HDM up to one-loop and in MSSM upto two-loops have been derived in [15–18,20]. The one-loop equations in both the theories are,

$$\begin{aligned} 16\pi^2 \frac{dv_u}{dt} &= \left[ \sum_i C_i^v g_i^2 - \text{Tr} \left( 3Y_U Y_U^\dagger \right) \right] v_u \\ 16\pi^2 \frac{dv_d}{dt} &= \left[ \sum_i C_i^v g_i^2 - \text{Tr} \left( 3Y_D Y_D^\dagger + Y_E Y_E^\dagger \right) \right] v_d \end{aligned} \quad (2.6)$$

The gauge couplings in the three models obey the well known one-loop RGEs,

$$16\pi^2 \frac{dg_i}{dt} = b_i g_i^3 \quad (2.7)$$

Two-loop contributions have been derived by a number of authors [15–18,21–27]. The coefficients appearing in the RHS of (2.4)–(2.7) are defined in the three different cases,

## SM, 2HDM

$$\begin{aligned}
C_i^u &= \left( \frac{17}{20}, \frac{9}{4}, 8 \right) \\
C_i^d &= \left( \frac{1}{4}, \frac{9}{4}, 8 \right) \\
C_i^e &= \left( \frac{9}{4}, \frac{9}{4}, 0 \right) \\
C_i^v &= \left( \frac{9}{20}, \frac{9}{4}, 0 \right)
\end{aligned} \tag{2.8}$$

## MSSM

$$\begin{aligned}
C_i^u &= \left( \frac{13}{5}, 3, \frac{16}{3} \right) \\
C_i^d &= \left( \frac{7}{15}, 3, \frac{16}{3} \right) \\
C_i^e &= \left( \frac{9}{5}, 3, 0 \right) \\
C_i^v &= \left( \frac{3}{20}, \frac{3}{4}, 0 \right)
\end{aligned} \tag{2.9}$$

## SM

$$\begin{aligned}
b_i &= \left( \frac{41}{10}, -\frac{19}{6}, -7 \right) \\
(a, b, c) &= (1, 1, -1)
\end{aligned} \tag{2.10}$$

## 2HDM

$$\begin{aligned}
b_i &= \left( \frac{21}{5}, -3, -7 \right) \\
(a, b, c) &= \left( 0, 1, \frac{1}{3} \right)
\end{aligned} \tag{2.11}$$

## MSSM

$$\begin{aligned}
b_i &= \left( \frac{33}{5}, 1, -3 \right) \\
(a, b, c) &= \left( 0, 2, \frac{2}{3} \right)
\end{aligned} \tag{2.12}$$

For the sake of simplicity we have neglected the Yukawa interactions of neutrinos. Assuming that the right-handed neutrinos are massive ( $M_N > 10^{13}$  GeV) our formulas are valid below  $M_N$  to a very good approximations even if such interactions are included.

### III. RGES AND ANALYTIC FORMULAS FOR RUNNING MASSES

Using the definition (2.1) and (2.4)-(2.12), we obtain the RGEs for the mass matrices in the broken phases of SM, 2HDM, or MSSM in the following form,

$$\begin{aligned} 16\pi^2 \frac{dM_U}{dt} &= \left( -\sum_i C_i g_i^2 + \tilde{a} Y_U Y_U^\dagger + \tilde{b} Y_D Y_D^\dagger \right) M_U \\ 16\pi^2 \frac{dM_D}{dt} &= \left( -\sum_i C'_i g_i^2 + \tilde{b} Y_U Y_U^\dagger + \tilde{a} Y_D Y_D^\dagger \right) M_D \\ 16\pi^2 \frac{dM_E}{dt} &= \left( -\sum_i C''_i g_i^2 + \tilde{c} Y_E Y_E^\dagger \right) M_E \end{aligned} \quad (3.1)$$

where the coefficients in the RHS are defined for the three cases.

SM, 2HDM

$$\begin{aligned} C_i &= \left( \frac{2}{5}, 0, 8 \right) \\ C'_i &= \left( -\frac{1}{5}, 0, 8 \right) \\ C''_i &= \left( \frac{9}{5}, 0, 0 \right) \end{aligned} \quad (3.2)$$

MSSM

$$\begin{aligned} C_i &= \left( \frac{49}{20}, \frac{9}{4}, \frac{16}{3} \right) \\ C'_i &= \left( \frac{19}{60}, \frac{9}{4}, \frac{16}{3} \right) \\ C''_i &= \left( \frac{33}{20}, \frac{9}{4}, 0 \right) \\ (\tilde{a}, \tilde{b}, \tilde{c}) &= (3, 1, 3) \end{aligned} \quad (3.3)$$

SM

$$(\tilde{a}, \tilde{b}, \tilde{c}) = \left( \frac{3}{2}, -\frac{3}{2}, \frac{3}{2} \right) \quad (3.4)$$

2HDM

$$(\tilde{a}, \tilde{b}, \tilde{c}) = \left( \frac{3}{2}, \frac{1}{2}, \frac{3}{2} \right) \quad (3.5)$$

Defining the diagonal mass matrices  $\hat{M}_F$ , the diagonal Yukawa matrices  $(\hat{Y}_F)$  and the CKM matrix  $(V)$  through biunitary transformations  $L_F$  and  $R_F$  on the left(right)-handed fermion  $F_L(F_R)$  with  $F = U, D, E$

$$\begin{aligned}\hat{M}_F &= L_F^\dagger M_F R_F \\ \hat{Y}_F &= L_F^\dagger Y_F R_F \\ \hat{M}_F^2 &= L_F^\dagger M_F M_F^\dagger L_F \\ \hat{Y}_F^2 &= L_F^\dagger Y_F Y_F^\dagger L_F \\ V &= L_U^\dagger L_D\end{aligned}\tag{3.6}$$

and following the procedures outlined in [13,28] we obtain

$$\begin{aligned}\frac{d\hat{M}_U^2}{dt} &= [\hat{M}_U^2, L_U^\dagger \dot{L}_U] + \frac{1}{16\pi^2} \left[ -2 \sum_i C_i g_i^2 \hat{M}_U^2 + 2\tilde{a} \hat{Y}_U^2 \hat{M}_U^2 + \tilde{b} (V \hat{Y}_D^2 V^\dagger \hat{M}_U^2 + \hat{M}_U^2 V \hat{Y}_D^2 V^\dagger) \right] \\ \frac{d\hat{M}_D^2}{dt} &= [\hat{M}_D^2, L_D^\dagger \dot{L}_D] + \frac{1}{16\pi^2} \left[ -2 \sum_i C'_i g_i^2 \hat{M}_D^2 + 2\tilde{a} \hat{Y}_D^2 \hat{M}_D^2 + \tilde{b} (V^\dagger \hat{Y}_U^2 V \hat{M}_D^2 + \hat{M}_D^2 V^\dagger \hat{Y}_U^2 V) \right] \\ \frac{d\hat{M}_E^2}{dt} &= [\hat{M}_E^2, L_E^\dagger \dot{L}_E] + \frac{1}{16\pi^2} \left[ -2 \sum_i C''_i g_i^2 \hat{M}_E^2 + 2\tilde{c} \hat{Y}_E^2 \hat{M}_E^2 \right]\end{aligned}\tag{3.7}$$

where  $\dot{L}_F = \frac{dL_F}{dt}$ .

We point out that in the corresponding RGEs for Yukawa couplings given by eq. (2.13) in Ref. [28], the terms  $-2 \sum_i C_i^u g_i^2 \hat{Y}_U^2 / (16\pi^2)$ ,  $-2 \sum_i C_i^d g_i^2 \hat{Y}_D^2 / (16\pi^2)$  and  $-2 \sum_i C_i^e g_i^2 \hat{Y}_E^2 / (16\pi^2)$  are missing from the R.H.S.

The diagonal elements of  $L_F^\dagger \dot{L}_F (F = U, D, E)$  are made to vanish in the usual manner by diagonal phase multiplication. The nondiagonal elements of both sides of (3.7) give the same RGEs for CKM matrix elements as before which on integration yields [28,29],

$$|V_{\alpha\beta}(\mu)| = \begin{cases} |V_{\alpha\beta}(m_t)| e^{-\frac{3}{2}c(I_t(\mu) + I_b(\mu))}, & \alpha\beta = ub, cb, tb, ts \\ |V_{\alpha\beta}(m_t)|, & \text{otherwise} \end{cases}\tag{3.8}$$

Taking diagonal elements of both sides of (3.7) and using dominance of Yukawa couplings of the third generation over the first two, except the charm quark, we obtain RGEs for the mass eigen values of quarks and leptons,

$$\begin{aligned}
16\pi^2 \frac{dm_u}{dt} &= \left[ - \sum_i C_i g_i^2 + \tilde{b} y_b^2 |V_{ub}|^2 \right] m_u \\
16\pi^2 \frac{dm_c}{dt} &= \left[ - \sum_i C_i g_i^2 + \tilde{a} y_c^2 + \tilde{b} y_b^2 |V_{cb}|^2 \right] m_c \\
16\pi^2 \frac{dm_t}{dt} &= \left[ - \sum_i C_i g_i^2 + \tilde{a} y_t^2 + \tilde{b} y_b^2 |V_{tb}|^2 \right] m_t \\
16\pi^2 \frac{dm_j}{dt} &= \left[ - \sum_i C'_i g_i^2 + \tilde{b} y_t^2 |V_{tj}|^2 \right] m_j, j = d, s \\
16\pi^2 \frac{dm_b}{dt} &= \left[ - \sum_i C'_i g_i^2 + \tilde{a} y_b^2 + \tilde{b} y_t^2 |V_{tb}|^2 \right] m_b \\
16\pi^2 \frac{dm_j}{dt} &= \left[ - \sum_i C''_i g_i^2 \right] m_j, j = e, \mu \\
16\pi^2 \frac{dm_\tau}{dt} &= \left[ - \sum_i C''_i g_i^2 + \tilde{c} y_\tau^2 \right] m_\tau
\end{aligned} \tag{3.9}$$

Integrating (3.9) and using the corresponding low-energy values, the new analytic formulas are obtained in the small mixing limit as

$$\begin{aligned}
m_u(\mu) &= m_u(1 \text{ GeV}) \eta_u^{-1} B_u^{-1} \\
m_c(\mu) &= m_c(m_c) \eta_c^{-1} B_u^{-1} e^{\tilde{a} I_c} \\
m_t(\mu) &= m_t(m_t) B_u^{-1} e^{\tilde{a} I_t + \tilde{b} I_b} \\
m_i(\mu) &= m_i(1 \text{ GeV}) \eta_i^{-1} B_d^{-1}, i = d, s \\
m_b(\mu) &= m_b(m_b) \eta_b^{-1} B_d^{-1} e^{\tilde{a} I_b + \tilde{b} I_t} \\
m_i(\mu) &= m_i(1 \text{ GeV}) \eta_i^{-1} B_e^{-1}, i = e, \mu \\
m_\tau(\mu) &= m_\tau(m_\tau) \eta_\tau^{-1} B_e^{-1} e^{\tilde{c} I_\tau}
\end{aligned} \tag{3.10}$$

where

$$\begin{aligned}
B_u &= \prod \left( \frac{\alpha_i(\mu)}{\alpha_i(m_t)} \right)^{\frac{C_i}{2b_i}} \\
B_d &= \prod \left( \frac{\alpha_i(\mu)}{\alpha_i(m_t)} \right)^{\frac{C'_i}{2b_i}} \\
B_e &= \prod \left( \frac{\alpha_i(\mu)}{\alpha_i(m_t)} \right)^{\frac{C''_i}{2b_i}}
\end{aligned} \tag{3.11}$$

$$I_f(\mu) = \frac{1}{16\pi^2} \int_{\ln m_t}^{\ln \mu} y_f^2(t') dt' \quad (3.12)$$

The ratio  $\eta_f (f = u, d, c, s, b, e, \mu, \tau)$  appearing in (3.10) is the QCD-QED rescaling factor for fermion mass  $m_f$ .

Comparison with earlier and recent derivation of analytic formulas [30,31] shows several differences. Whereas the top-quark Yukawa coupling integral in (3.12) has been predicted to affect the running of  $m_u(\mu)$  and  $m_c(\mu)$  our formulas predict no such dependence. Similarly, whereas the  $b$ -quark and the  $\tau$ -lepton Yukawa coupling integrals have been predicted to affect the running charged-lepton masses  $m_e(\mu)$  and  $m_\mu(\mu)$ , our formulas predict no such contributions. Our formulas predict that in all the three cases, SM, 2HDM, or MSSM, the third generation Yukawa couplings do not affect the running masses of the first two generations in the small mixing limit. Also for the third generation running masses, the Yukawa coupling integrals occur in the exponential factors with different coefficients as compared to [30,31]. The dependence on gauge couplings are also noted to be quite different in our analytic formulas. Whereas earlier derivation [30] predicted the occurrence of the exponents  $C_i^u/2b_i$ ,  $C_i^d/2b_i$ ,  $C_i^e/2b_i$  on the RHS of (3.11), our formulas predict the corresponding exponents to be  $C_i/2b_i$ ,  $C'_i/2b_i$ ,  $C''_i/2b_i$  respectively. Thus, the new analytic formulas derived here at one-loop level predict substantially new dependence on gauge and Yukawa couplings for the running masses. Our formulas for the case of MSSM are the same as those obtained in [13] where the SUSY scale was taken as  $M_S = m_t$ . As derived in [13] for the MSSM, the formula for running  $\tan \beta(\mu)$  has also the same form also in the 2HDM at one-loop level.

$$\tan \beta(\mu) = \tan \beta(m_t) \exp [-3I_t(\mu) + 3I_b(\mu) + I_\tau(\mu)] \quad (3.13)$$

In contrast, when running of VEVs are ignored, apart from the mass predictions being different,  $\tan \beta$  is also predicted to be the same for all values of  $\mu > M_Z$  in 2HDM or MSSM [30,31].

#### IV. FORMULAS IN MSSM FOR $M_S > M_Z$

In the MSSM the natural SUSY scale ( $M_S$ ) could be very different from the weak scale with  $M_S \approx O$  (TeV), whereas  $M_S \gg 1$  TeV has gauge hierarchy problem. As our new contribution in MSSM in this paper, compared to [13], we present new analytic formulas for all charged fermion masses for any SUSY scale  $M_S > M_Z$  by running them from  $m_t - M_S$  as in SM and then from  $M_S - \mu$  as in MSSM.

$$m_u(\mu) = m_u(1 \text{ GeV}) \eta_u^{-1} G_u(\mu) \quad (4.1)$$

$$m_c(\mu) = m_c(m_c) \eta_c^{-1} G_u(\mu) \exp\left(\frac{3}{2} I_c(M_S) + 3 \tilde{I}_c(\mu)\right) \quad (4.2)$$

$$m_t(\mu) = m_t(m_t) G_u(\mu) \exp\left(\frac{3}{2} I_t(M_S) - \frac{3}{2} I_b(M_S) + 3 \tilde{I}_t(\mu) + \tilde{I}_b(\mu)\right) \quad (4.3)$$

$$m_i(\mu) = m_i(1 \text{ GeV}) \eta_i^{-1} G_d(\mu), i = d, s \quad (4.4)$$

$$m_b(\mu) = m_b(m_b) \eta_b^{-1} G_d(\mu) \exp\left(\frac{3}{2} I_b(M_S) - \frac{3}{2} I_t(M_S) + 3 \tilde{I}_b(\mu) + \tilde{I}_t(\mu)\right) \quad (4.5)$$

$$m_i(\mu) = m_i(1 \text{ GeV}) \eta_i^{-1} G_e(\mu), i = e, \mu \quad (4.6)$$

$$m_\tau(\mu) = m_\tau(m_\tau) \eta_\tau^{-1} G_e(\mu) \exp\left(\frac{3}{2} I_\tau(M_S) + 3 \tilde{I}_\tau(\mu)\right) \quad (4.7)$$

where

$$\begin{aligned} G_u(\mu) &= \left(\frac{\alpha_1(M_S)}{\alpha_1(m_t)}\right)^{\frac{-2}{41}} \left(\frac{\alpha_3(M_S)}{\alpha_3(m_t)}\right)^{\frac{4}{7}} \left(\frac{\alpha_1(\mu)}{\alpha_1(M_S)}\right)^{\frac{-49}{264}} \left(\frac{\alpha_2(\mu)}{\alpha_2(M_S)}\right)^{\frac{-9}{8}} \left(\frac{\alpha_3(\mu)}{\alpha_3(M_S)}\right)^{\frac{8}{9}} \\ G_d(\mu) &= \left(\frac{\alpha_1(M_S)}{\alpha_1(m_t)}\right)^{\frac{1}{41}} \left(\frac{\alpha_3(M_S)}{\alpha_3(m_t)}\right)^{\frac{4}{7}} \left(\frac{\alpha_1(\mu)}{\alpha_1(M_S)}\right)^{\frac{-19}{792}} \left(\frac{\alpha_2(\mu)}{\alpha_2(M_S)}\right)^{\frac{-9}{8}} \left(\frac{\alpha_3(\mu)}{\alpha_3(M_S)}\right)^{\frac{8}{9}} \\ G_e(\mu) &= \left(\frac{\alpha_1(M_S)}{\alpha_1(m_t)}\right)^{\frac{-9}{41}} \left(\frac{\alpha_1(\mu)}{\alpha_1(M_S)}\right)^{\frac{-1}{8}} \left(\frac{\alpha_2(\mu)}{\alpha_2(M_S)}\right)^{\frac{-9}{8}} \end{aligned} \quad (4.8)$$

$$\tilde{I}_f(\mu) = \frac{1}{16\pi^2} \int_{\ln M_S}^{\ln \mu} y_f^2(t') dt' \quad (4.9)$$

and  $I_f(M_S)$  is defined through (3.12) with  $\mu = M_S$ . Running of the elements of the CKM matrix in the MSSM are modified by the following formulas

$$|V_{\alpha\beta}(\mu)| = \begin{cases} |V_{\alpha\beta}(m_t)| e^{\frac{3}{2}(I_t(M_S) + I_b(M_S))} e^{-(\tilde{I}_t(\mu) + \tilde{I}_b(\mu))}, & \alpha\beta = ub, cb, tb, ts \\ |V_{\alpha\beta}(m_t)|, & \text{otherwise} \end{cases} \quad (4.10)$$

The one-loop formula for  $\tan \beta(\mu)$  in (3.13) is also modified,

$$\tan \beta(\mu) = \tan \beta(M_S) \exp \left( -3\tilde{I}_t(\mu) + 3\tilde{I}_b(\mu) + \tilde{I}_\tau(\mu) \right) \quad (4.11)$$

The analytic formulas (4.1)-(5.1) hold good for any value of  $m_t < M_S < \mu$ . It may be noted that in the limit of  $M_S \rightarrow m_t$ ,  $I_f(M_S) \rightarrow 0$ ,  $\tilde{I}_f(\mu) \rightarrow I_f(\mu)$  and the formulas (4.1)-(4.11) reduce to those obtained in [13]. It is to be noted that corresponding exponent in the expression  $B_u$  in eq. (3.6) of ref. [13] should be corrected as  $\frac{49}{264}$  in place of  $\frac{43}{792}$ .

## V. NUMERICAL PREDICTIONS AT HIGHER SCALES

The analytic formulas given in the previous section predict masses and the CKM matrix elements upto one-loop level at higher scales. We have also estimated numerically the effect of scale dependent VEVs on predictions of the running masses at two-loop level. We solve RGEs for the Yukawa matrices and VEVs including two loop contributions in SM and MSSM [15–18,25–27] numerically and obtain the mass matrices at higher scales from the corresponding products of the two. For this purpose the elements of the CKM matrix at higher scales have been obtained by running them through one-loop RGEs given by (3.8) with appropriate values of the coefficient  $c$  given in (2.10)-(2.12) [28,29]. In 2HDM we carry out all numerical estimations at one-loop level. We use the following inputs for the running masses ( $m_i$ ), SM gauge couplings ( $\alpha_1, \alpha_2, \alpha_3$ ), electromagnetic finestructure constant( $\alpha$ ), electroweak mixing angle and the CKM matrix ( $V$ ) at  $\mu = M_Z$  which have been obtained from the experimental data [32].

$$\begin{aligned} m_u &= 2.33_{-0.45}^{+0.42} \text{ MeV}, m_c = 677_{-61}^{+56} \text{ MeV}, \\ m_t &= 181 \pm 13 \text{ GeV}, m_d = 4.69_{-0.66}^{+0.60} \text{ MeV}, \\ m_s &= 93.4_{-13.0}^{+11.8} \text{ MeV}, m_b = 3.00 \pm 0.11 \text{ GeV}, \\ m_e &= 0.48684727 \pm 0.00000014 \text{ MeV}, \\ m_\mu &= 102.75138 \pm 0.00033 \text{ MeV}, \\ m_\tau &= 1.74669_{-0.00027}^{+0.00030} \text{ GeV} \end{aligned} \quad (5.1)$$

$$\begin{aligned}
\alpha_1(M_Z) &= 0.016829 \pm 0.000017 \\
\alpha_2(M_Z) &= 0.033493^{+0.000042}_{-0.000038} \\
\alpha_3(M_Z) &= 0.118 \pm 0.003 \\
\alpha_{em}^{-1} &= 128.896 \pm 0.09 \\
\sin^2 \theta_W &= 0.23165 \pm 0.000024
\end{aligned} \tag{5.2}$$

$$V(M_Z) = \begin{pmatrix} 0.9757, & 0.2205, & 0.0030e^{-i\delta} \\ -0.2203 - 0.0001e^{i\delta}, & 0.9747, & 0.0373 \\ 0.0082 - 0.0029e^{i\delta}, & -0.0364 - 0.0007e^{i\delta}, & 0.9993 \end{pmatrix} \tag{5.3}$$

For the sake of convince we have used  $\delta = \pi/2$  as in [32]. The choice of same inputs enables us to compare our results on mass predictions with those obtained with scale-independent assumption on the VEVs in SM and MSSM [32]. We neglect mixings among charged-leptons and use the diagonal basis for up-quarks.

The variations of VEVs as a function of  $\mu$  are shown in Figs. 1-2 for the SM, 2HDM, and MSSM where the initial value of  $\tan \beta(M_Z) = 10$  has been used for the latter two cases. In these and certain other Figs. we have used the variable  $t = \ln \mu$  along X-axis where  $\mu$  is in units of GeV. It is quite clear that in the SM as well as the other cases the running effects of the VEVs contribute to very significant departures from the assumed scale-independent values [28,30–32]. Thus the predicted running masses are to be different in all the three cases. Since  $v_u(\mu)$  increases and  $v_d(\mu)$  decreases with increasing  $\mu$ , the up-quark masses are expected to have decreasing effects whereas the down-quark and charged-lepton masses are expected to have increasing effects at higher scales in MSSM and 2HDM. But in the SM all the masses are expected to have decreasing effects due to decreasing value of  $v(\mu)$ . In fact these features are clearly exhibited in all numerical values of mass predictions carried out in this investigation. It is to be noted that almost all fermion masses, except the top-quark,  $b$ -quark and the  $\tau$ -lepton near the perturbative limits, decrease at higher scales due to decrease in corresponding Yukawa couplings. But the effect of running VEVs contribute to additional decreasing or increasing factors in the respective cases.

The predictions of all the charged fermion masses as a function of  $t = \ln \mu$  are shown in Fig. 3 with  $M_S = M_Z$  and in Fig. 4 with  $M_S = 1$  TeV in the case of MSSM using  $\tan \beta(M_Z) = 10$ . The corresponding predictions in 2HDM and SM are shown in Figs. 5–6. In Fig. 7 we display the comparison of mass predictions as functions of  $t = \ln \mu$  with and without running VEVs in MSSM assuming  $M_S = M_Z$  and  $\tan \beta = 10$ . Although the differences in the two types of predictions are clearly distinguishable, they are quite prominent in the up-quark sectors. While the new contributions are seen to be significant for the down-quarks and charged-leptons at higher scales with  $\mu \geq 10^7$  GeV, in the case of up-quarks the contributions are found to be important starting from  $\mu = O$  (TeV). As compared to the scale-independent assumption [32], our predictions are clearly smaller for the up-quarks and larger for the down-quarks and charged-leptons as indicated by solid-line curves in Fig. 7. With the input values for  $m_t$  and  $m_b$  in (5.1), the lowest allowed value of  $\tan \beta(M_S)$  is determined by observing the perturbative limit for the top-quark Yukawa coupling at the GUT scale,  $y_t^2(M_{GUT})/4\pi \leq 1.0$  and the highest allowed value of  $\tan \beta(M_S)$  is determined from the corresponding limit on the  $b$ -quark Yukawa coupling.

### MSSM

$$M_S = M_Z : 2.3_{-0.6}^{+4.8} \leq \tan \beta(M_S) \leq 58.7_{-2.0}^{+3.4}, \quad (5.4)$$

$$M_S = 1 \text{ TeV} : 1.7_{-0.4}^{+1.3} \leq \tan \beta(M_S) \leq 64.8_{-4.3}^{+3.6} \quad (5.5)$$

The allowed region for  $\tan \beta(M_S)$  as a function of  $M_S$  in MSSM is shown in Fig. 8 where the solid (dashed) lines are due to the central values (uncertainties) in the inputs of  $m_t$  and  $m_b$ . It is clear that the allowed region for  $\tan \beta$  increases, although slowly, with increasing  $M_S$ . In the 2HDM the allowed region for  $\tan \beta$  is found to be substantially larger.

### 2HDM

$$1.2_{-0.2}^{+0.3} \leq \tan \beta(M_Z) \leq 68.9 \pm 2.7 \quad (5.6)$$

We have noted that in all the three effective theories the difference between one- and two-loop estimation of running masses at the highest scale ( $M_U$ ) varies between 1-5%, the

lowest discrepancy being for the leptons and the highest being for the top-quark. But in MSSM and 2HDM this discrepancy increases to 10-12% for the  $b$ - and the top-quarks as the respective perturbative limits are approached.

The running VEVs in MSSM and 2HDM lead to variation of  $\tan\beta(\mu)$  as a function of  $\mu$  over its initial value at  $M_Z$ . This is shown in Fig. 9 for different input values where the dashed (solid) line represents the case for 2HDM (MSSM). In both the theories  $\tan\beta(\mu)$  decreases (increases) from its initial value when the latter crosses a critical point. This critical value is  $\tan\beta(M_Z) \approx 56$  (52) in MSSM (2HDM). In Fig. 10 we present  $\tan\beta(M_U)$  at the GUT scale as a function of  $\tan\beta(M_Z)$  for both the theories. We observe steep rise in the curves as the respective perturbative limits are approached in the large  $\tan\beta(M_Z)$  region.

Using the central values of  $m_t(M_Z)$ ,  $m_b(M_Z)$  and  $m_\tau(M_Z)$  from (5.1), we have studied variation of  $m_t(\mu)$ ,  $m_b(\mu)$  and  $m_\tau(\mu)$  for different values of  $\mu = 10^9$  GeV,  $10^{13}$  GeV and  $2 \times 10^{16}$  GeV each as a function of various low-energy input values of  $\tan\beta(M_Z)$  in MSSM and 2HDM. These results are presented in Figs. 11-13 for the 2HDM (dashed lines) and for the MSSM (solid lines) with  $M_S = M_Z$ . It is clear that the perturbatively allowed range of  $\tan\beta$  decreases with increasing  $\mu$  both for MSSM and 2HDM.

We have examined simultaneous variation of  $m_t(\mu)$  as a function of  $\mu$  and  $\tan\beta(\mu)$  which is displayed in the three dimensional plot of Fig. 14 for the input value of  $m_t(M_Z) = 181$  GeV and  $\tan\beta(M_S) = 2 - 58$ , where  $M_S = 1$  TeV. Using the top-quark mass at  $\mu = M_Z$ , we have calculated  $m_t(\mu)$  and  $\tan\beta(\mu)$  at every  $\mu$  between  $M_S - M_U$  for input value of  $\tan\beta(M_S) = 2 - 59$ . The results are displayed in the three dimensional plot. The variations of the running mass predictions at the GUT scale ( $M_U = 2 \times 10^{16}$  GeV) as a function of the SUSY scale ( $M_S = M_Z - 10^4$  GeV) are shown in Figs. 15-16 for the third generation fermions using various input values of  $\tan\beta(M_S)$ . We find that the top-quark mass at the GUT scale at first decreases sharply in the smaller and larger  $\tan\beta$  regions as  $M_S$  increases and then remains almost constant for  $M_S = 3 \times 10^3 - 10^4$  GeV. Similarly the predicted  $b$  and  $\tau$  masses decrease with increasing  $M_S$  although fall off is slower in the case of  $\tau$  in the

large  $\tan\beta$  region.

Numerical values of predictions of the running masses are presented in Table. I for the SM at three different scales  $\mu = 10^9$  GeV,  $10^{13}$  GeV and  $2 \times 10^{16}$  GeV. The two-loop contributions to the RG evolution of Yukawa couplings depends, although very weakly, upon the Higgs quartic couplings  $\lambda$  which is related to the Higgs mass ( $M_H$ ) and VEV( $v$ ),  $\lambda = M_H^2/v^2$ . We have used the two-loop RGEs for  $\lambda(\mu)$  for the SM [16] and evaluated the running masses and VEVs of Table. I for the input value of the Higgs mass  $M_H = 250$  GeV. Changing the Higgs mass in the allowed range of  $M_H = 220 - 260$  GeV [32] does not change the results of Table. I significantly. The uncertainties in the quantities are due to those in the running masses at  $\mu = M_Z$ . The mass matrices for  $M_b(\mu)$  and  $M_u(\mu)$  are modified by the factors  $v(\mu)/v(M_Z)$  where  $v(M_Z) \approx 174$  GeV and the CKM matrices at higher scales remain the same as in [32]. The computed values of masses are found to be less when compared to those obtained with scale-independent assumption [32]. This is clearly understood as the running VEV in the SM decreases with increasing  $\mu$ . For example in the SM at  $\mu = 2 \times 10^{16}$  GeV our predictions are  $(m_u, m_c, m_t) = (0.83 \text{ MeV}, 242.6 \text{ MeV}, 75.4 \text{ GeV})$  as compared to [32]  $(m_u, m_c, m_t) = (0.94 \text{ MeV}, 272 \text{ MeV}, 84 \text{ GeV})$  where the running effect on the VEV has been ignored. In Tables II-III numerical values of masses, VEVs and  $\tan\beta$  are given at the same three scales for the MSSM and 2HDM with  $\tan\beta(M_Z) = 10$  and 55 in each of the two theories. As emphasized in this paper our high scale estimations predict quite significantly different values for the running masses especially in the up-quark sector. Although the CKM matrices at high scales remain the same as in scale-independent assumptions on the VEVs, the up-quark mass matrices are modified by the factor  $v_u(\mu)/v_u(M_S)$ , but the down-quark and charged-lepton mass matrices are modified by the factor  $v_d(\mu)/v_d(M_S)$ . In the MSSM with  $M_S = M_Z$  and  $\tan\beta(M_Z) = 10$  including running effect of the VEVs, the GUT scale predictions are  $(m_u, m_c, m_t) = (0.70 \text{ MeV}, 200 \text{ MeV}, 73.5 \text{ GeV})$  as compared to [32]  $(m_u, m_c, m_t) = (1.04 \text{ MeV}, 302 \text{ MeV}, 129 \text{ GeV})$ . But by increasing the SUSY scale to  $M_S = 1$  TeV and in the large  $\tan\beta$ -region we find substantial decrease in the predicted values of the top-quark mass at the GUT scale leading to  $(m_u, m_c, m_t) = (0.72 \text{ MeV}, 210$

MeV, 95.1 GeV). This is understood by noting that  $\tan \beta \approx 55$  is closer to the perturbative limit for which the top-quark Yukawa coupling is larger. Similarly from Table. II we note nearly a 20% increase in the  $m_b(M_U)$  with the increase of  $\tan \beta$  from 10 – 55. Similar effects are also noted in 2HDM as can be seen from Table. III where  $m_t$  decreases by nearly 7% as  $\tan \beta$  increases from 10 – 55. For larger effect the increase has to be larger in  $\tan \beta$  since the perturbative limit in this case is closer to  $\tan \beta \approx 69$  as compared to the MSSM case where the limiting value is  $\tan \beta \approx 59$ .

## VI. SUMMARY AND CONCLUSION

In the frequently exploited renormalisation scheme in gauge theories, the Yukawa couplings and VEVs in the SM, 2HDM and MSSM run separately [15–29]. The effect of scale dependence of the VEVs has been ignored while deriving analytic formulas [28,30,31] and obtaining numerical predictions at higher scales for the running masses of fermions [32], but appropriately taken into account more recently [13]. In this paper we have derived new analytic formulas in the SM and 2HDM and generalised the formulas of [13] for any supersymmetry scale ( $M_S > M_Z$ ). The new formulas exhibit substantially different functional dependence on gauge and Yukawa couplings in all the three effective theories. In particular, the running masses of the first two generations are found to be independent of Yukawa couplings of the third generations in the small mixing limit. Numerical predictions at two-loop level shows that all the running masses in the SM and only the up-quark masses in the MSSM and 2HDM decrease at high scales when compared with the predictions taking scale-independent VEVs. But in the case of MSSM and 2HDM the down-quark and the charged-lepton masses increase over the corresponding predictions obtained with scale-independent assumptions on the VEVs. Compared to the MSSM the perturbatively allowed region of  $\tan \beta$  is larger in 2HDM. In MSSM the allowed region shows a slow increase with the SUSY scale. We have also made predictions of the running masses at the GUT scale as a function of supersymmetry scale exhibiting new behaviours. We suggest that these new

analytic formulas and improved estimations on the running masses and  $\tan\beta$  at high scales be used as inputs to test models proposing unified explanations of quark-lepton masses.

#### **ACKNOWLEDGMENTS**

One of us (M.K.P.) thanks Professor E.A. Paschos for hospitality at the Institute of Physics, Dortmund University and Professor Y. Achiman for hospitality at the Institute of Physics, Wuppertal University where a part of this work was carried out. The authors thank Professor L. Lavoura for a useful comment. This work is supported by the DAE project No. 98/37/91/BRNS-Cell/731 of the Govt. of India.

## REFERENCES

- [1] J. Ellis, S. Kelly, D.V. Nanopoulos, Phys. Lett. **B260**, 131 (1991); U. Amaldi, W. de. Boer, H. Furstenau, Phys. Lett. **B260**, 447 (1991); P. Langacker, M. Luo, Phys. Rev. **D44**, 817 (1991).
- [2] D. G. Lee, R.N. Mohapatra, M.K. Parida, M. Rani, Phys. Rev. **D49**, 3704 (1995); N. G. Deshpande, E. Keith, P. Pal, Phys. Rev. **D46**, 2261 (1992); M. Rani, M.K. Parida, Phys. Rev. **D49**, 3704 (1994).
- [3] T.G. Rizzo, Phys. Rev. **D45**, 3903 (1992); F. Anselmo et. al. Nuovo Cim. **104A**, 1817 (1992); H. Murayama, T. Yanagida, Tohoku University Report No. TU-370, (1991) (Unpublished); U. Amaldi, W. de. Boer, P.H. Frampton, H. Furstenau, J.T. Liu, Phys. Lett. **B281**, 374 (1992); M.L. Kynshi, M.K. Parida, Phys. Rev. **D49**, 3711 (1994).
- [4] M.L. Kynshi, M.K. Parida, Phys. Rev. **D47**, R4830 (1993).
- [5] L. Lavoura and L. Wolfenstein, Phys. Rev. **D48**, 264 (1993).
- [6] M.K. Parida, A. Usmani, Phys. Rev. **D54**, 3663 (1996).
- [7] Y. Nomura, T. Sugimoto, hep-ph/9903334; Y. Nomura, T. Yanagida, Phys. Rev. **D59**, 017303 (1999); M. Fukugita, M. Tanimoto, T. Yanagida, hep-ph/9903484; M. Fukugita, M. Tanimoto, T. Yanagida, Phys. Rev. **D57**, 4429 (1998), hep-ph/9809554.
- [8] J.C. Pati, A. Salam, Phys. Rev. **D10**, 275 (1974).
- [9] H. Georgi, C. Jarlskog, Phys. Lett. **B86**, 297 (1979); J. Harvey, P. Ramond, D.B. Reiss, Phys. Lett. **B92**, 309 (1980); Nucl. Phys. **B119**, 223 (1982).
- [10] K.S. Babu, R.N. Mohapatra, Phys. Rev. Lett. **70**, 2845 (1993); D. G. Lee, R.N. Mohapatra, Phys. Rev. **D49**, 1353 (1995).
- [11] B. Brahmachari, R.N. Mohapatra, Phys. Rev. **D57**, 015003 (1998); K. Oda, E. Takasugi, M. Tanaka, M. Yoshimura, Phys. Rev. **D59**, 055001 (1999); K.S. Babu, J.C. Pati,

F. Wilczek, Nucl. Phys. **B566**, 33, (2000), hep-ph/9812538.

[12] G.A. Blair, W. Porod, P.M. Zerwas, hep-ph/0007107.

[13] M.K. Parida, B. Purkayastha, Eur. Phys. J. **C14**, 159 (2000), hep-ph/9902374.

[14] A. Sirlin, R. Zucchini, Nucl. Phys. **B266**, 389 (1986); A. Sirlin, Phys. Rev. **D22**, 971 (1980); W. Marciano, A. Sirlin, Phys. Rev. **D29**, 89 (1984); W. Marciano, A. Sirlin, Phys. Rev. **D22**, 2695 (1980); Phys. Rev. Lett. **46**, 163 (1981); S. Sarantakos, A. Sirlin, W. Marciano, Nucl. Phys. **B217**, 84 (1983); A. Sirlin, Phys. Lett. **B232**, 123 (1989); S. Fanchiotti, A. Sirlin, Phys. Rev. **D41**, 319 (1990).

[15] H. Arason, D.J. Castano, B. Keszthelyi, S. Mikaelian, E.J. Pirad, P. Ramond, B.D. Wright, Phys. Rev. Lett. **67**, 2933 (1991).

[16] H. Arason, D.J. Castano, B. Keszthelyi, S. Mikaelian, E.J. Pirad, P. Ramond, B.D. Wright, Phys. Rev. **D46**, 3945 (1992); H. Arason, D.J. Castano, E.J. Pirad, P. Ramond, Phys. Rev. **D47**, 232 (1993).

[17] D.J. Castano, E.J. Pirad, P. Ramond, Phys. Rev. **D49**, 4882 (1994).

[18] M.E. Machacek, M.T. Vaughn, Nucl. Phys. **B222**, 83 (1983); Nucl. Phys. **B236**, 221 (1984); Nucl. Phys. **B249**, 70 (1985).

[19] C. Balzleit, T. Hansmann, T. Mannel, B. Plumber, hep-ph/9810350.

[20] G. Cvetic, S.S. Hwang, C.S. Kim, Phys. Rev. **D58**, 116003 (1998); G. Cvetic, S.S. Hwang, C.S. Kim, hep-ph/9706323; Acta. Phys. Pol. **B28**, 2515 (1997).

[21] W. Grimus, Lett. Nuovo Cim. **27**, 169 (1979).

[22] B. Pendleton, G.G. Ross, Phys. Lett. **B98**, 296 (1981).

[23] K.S. Babu, C.N. Leung, J. Pantaleone, Phys. Lett. **B319**, 191 (1993).

[24] A.J. Buras, J. Ellis, M.K. Gaillard, D.V. Nanopoulos, Nucl. Phys. **B135**, 66 (1978);

D.V. Nanopoulos, D.A. Ross, Nucl. Phys. **B147**, 273 (1979).

[25] V. Barger, M.S. Barger, P. Ohmann, Phys. Rev. **D47**, 333 (1993).

[26] B. Grzadkowski, M. Lindner, Phys. Lett. **B193**, 71 (1987); B. Grzadkowski, M. Lindner, S. Theisen, Phys. Lett. **B198**, 64 (1987); M. Olechowski, S. Pokorski, Phys. Lett. **B257**, 388 (1991).

[27] N.K. Falk, Z. Phys. **C30**, 247 (1986).

[28] S.G. Naculich, Phys. Rev. **D48**, 5293 (1993).

[29] K. Sasaki, Z. Phys. **C32**, 146 (1996); K.S. Babu, Z. Phys. **C35**, 69 (1987); P. Binetruy, P. Ramond, Phys. Lett. **B350**, 49 (1995); K. Wang, Phys. Rev. **D54**, 5750 (1996).

[30] N.G. Deshpande, E. Keith, Phys. Rev. **D50**, 3513 (1994).

[31] P. Kielanowski, S.R. Juráez W., J.G. Mora H., Phys. Lett. **B479**, 181 (2000), hep-ph/0002062, S.R. Juráez W., S.F. Herrera H., P. Kielanowski, G. Mora H., hep-ph/0009148.

[32] H. Fusaoka, Y. Koide, Phys. Rev. **D57**, 3986 (1998).

## FIGURES

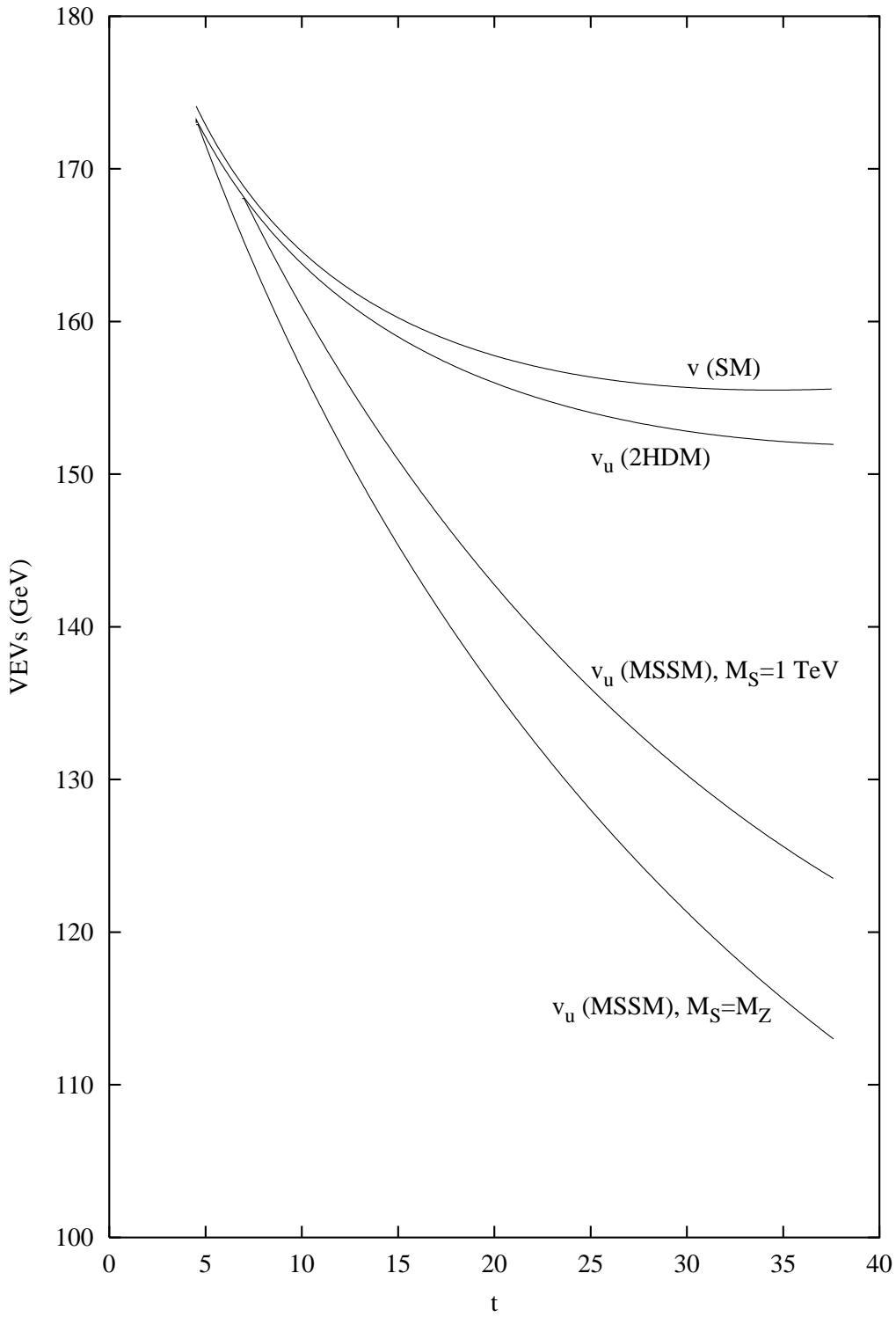


FIG. 1. Variation of running VEVs in the SM, 2HDM and MSSM as a function of  $\mu$  ( $t = \ln \mu$ ) showing substantial deviation from the scale-independent assumption.

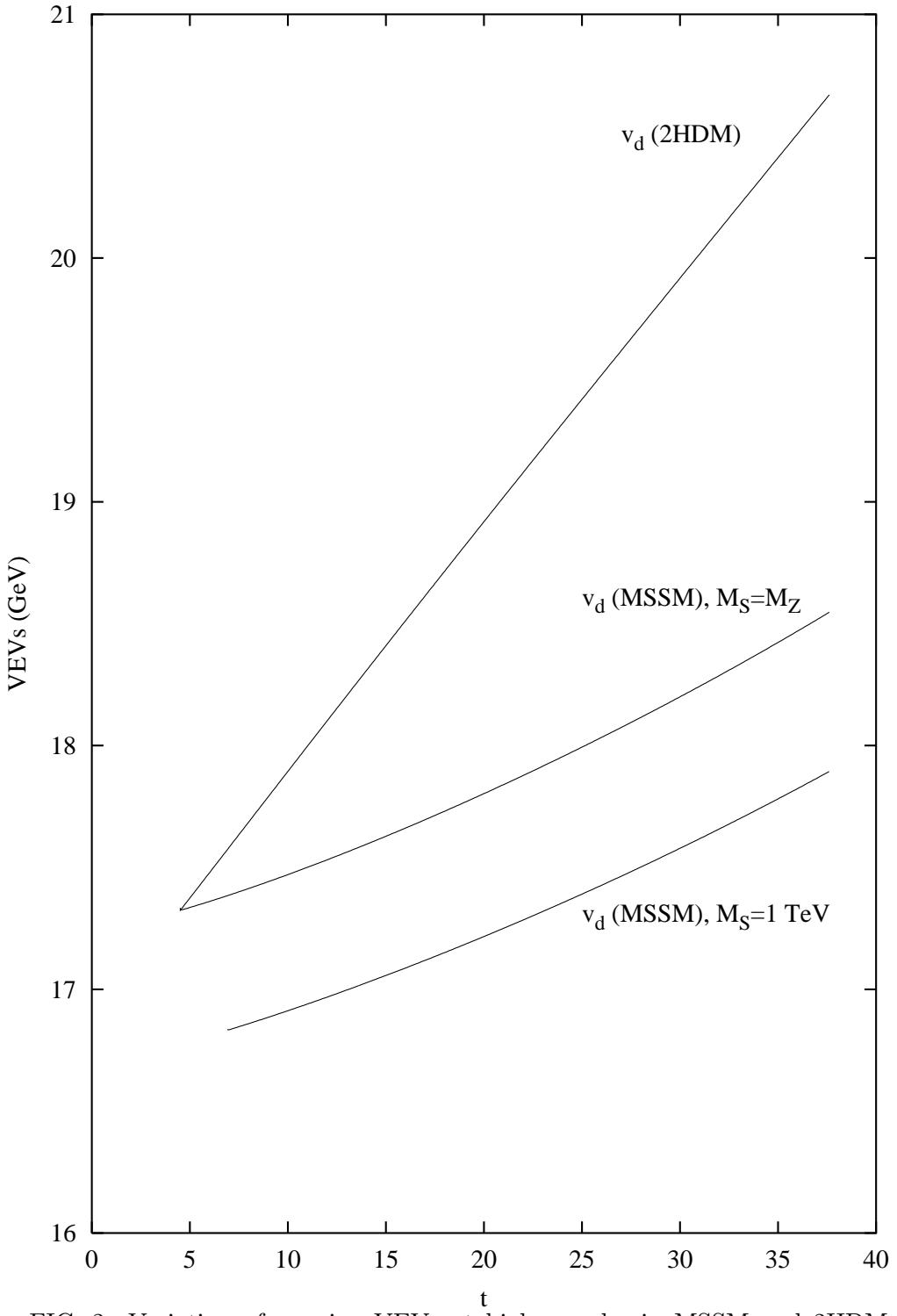


FIG. 2. Variation of running VEVs at higher scales in MSSM and 2HDM as a function of  $\mu(t = \ln \mu)$  showing substantial deviation from the scale-independent assumption.

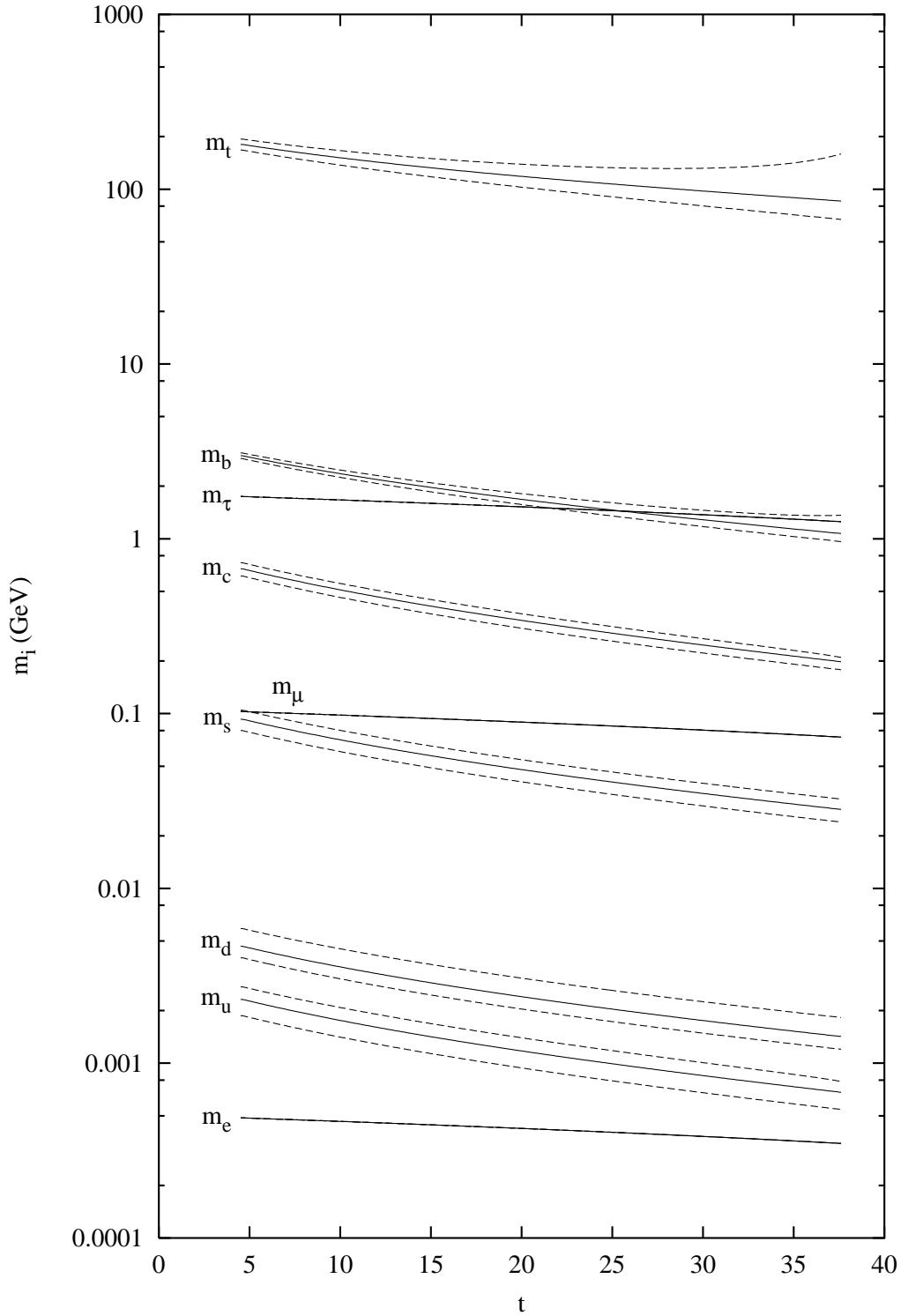


FIG. 3. Predictions of running masses at higher scales as a function of  $\mu$  ( $t = \ln \mu$ ) in MSSM with SUSY scale  $M_S = M_Z$  using the input parameters given in (5.1)-(5.3) and  $\tan \beta(M_S) = 10$ . The dashed lines are due to uncertainties in the input parameters.

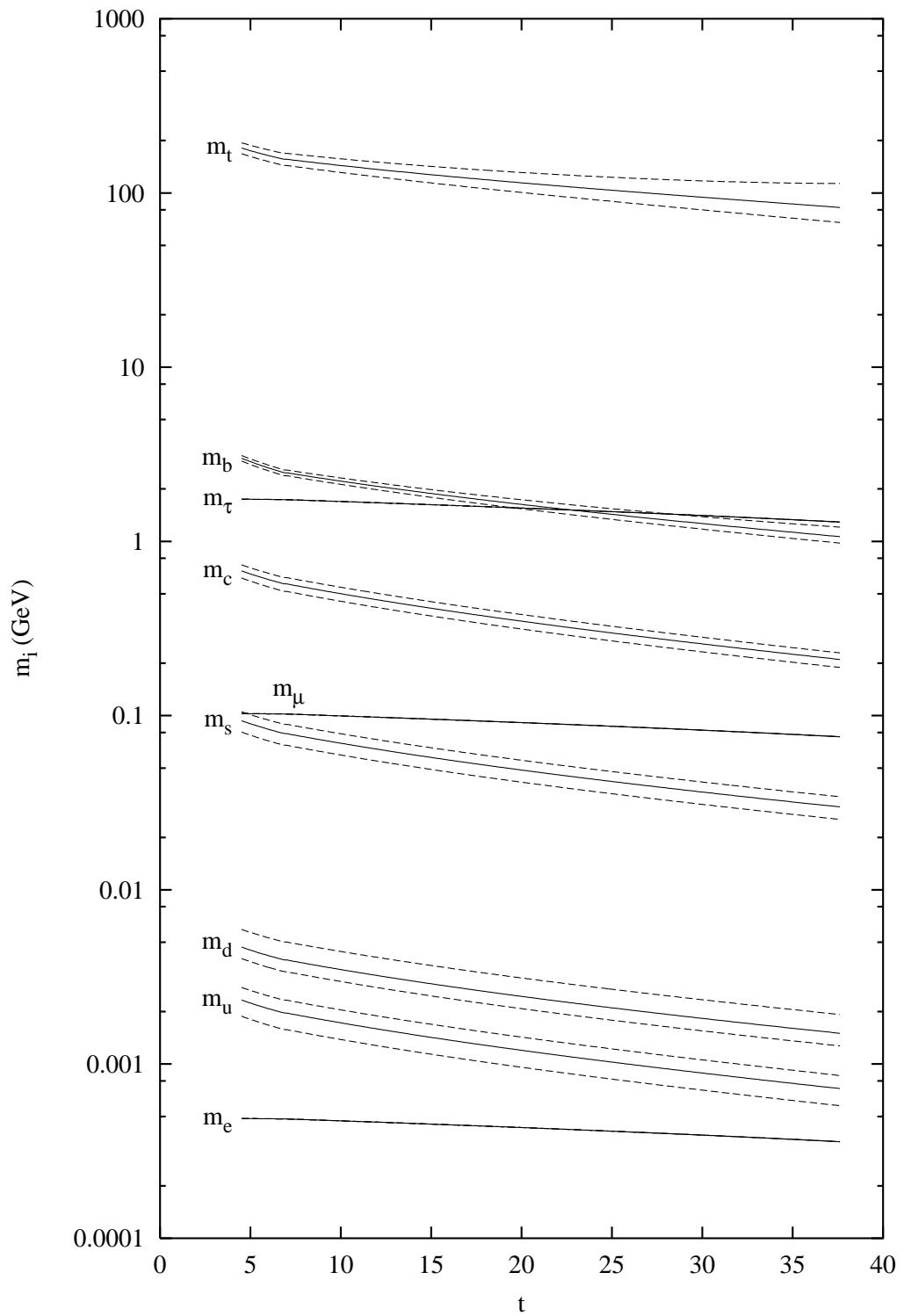


FIG. 4. Same as Fig. 3 but with  $M_S = 1$  TeV.

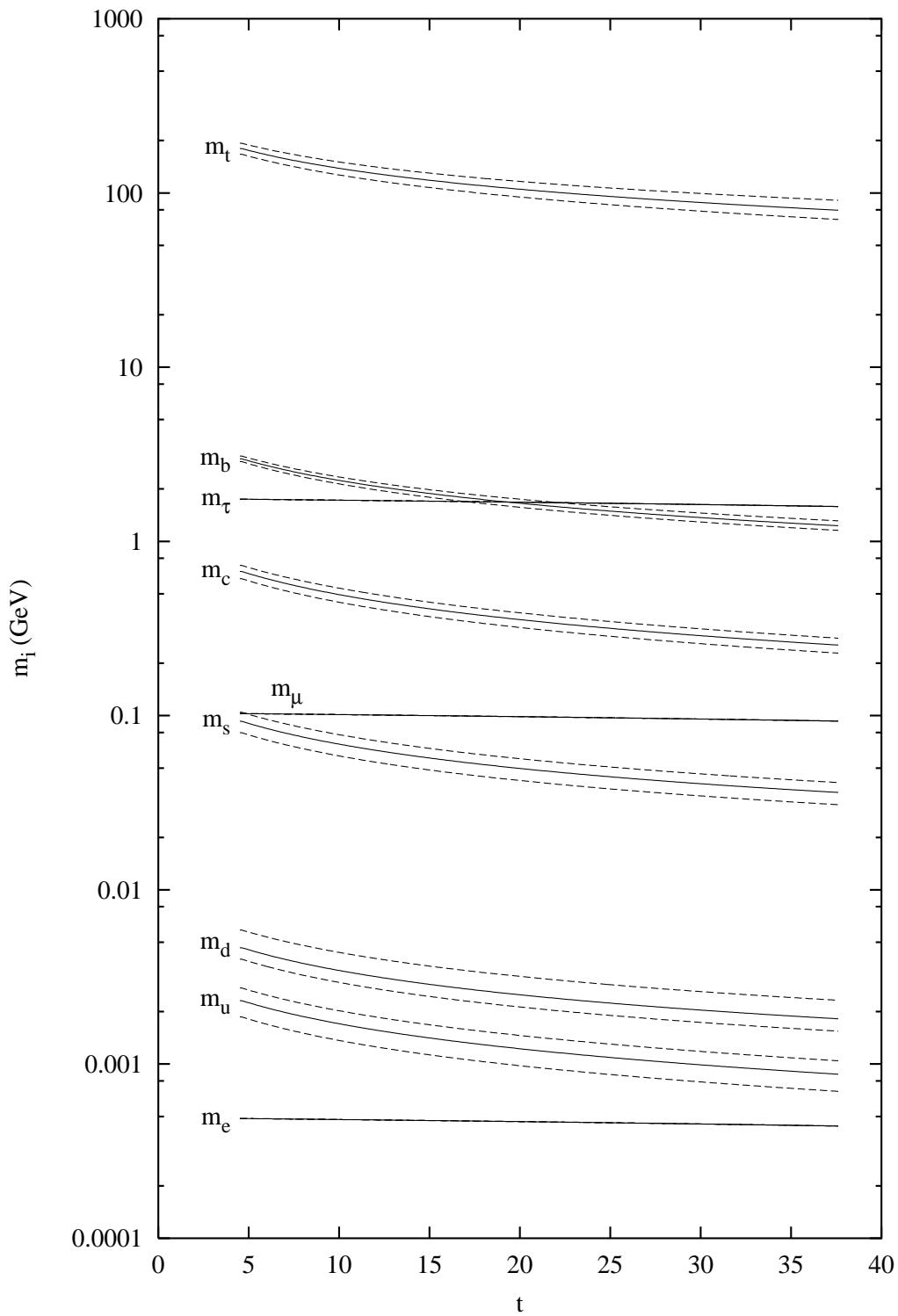


FIG. 5. Predictions of running masses at higher scales in the 2HDM using the input parameters given in (5.1)-(5.3) and  $\tan \beta(M_Z) = 10$ .

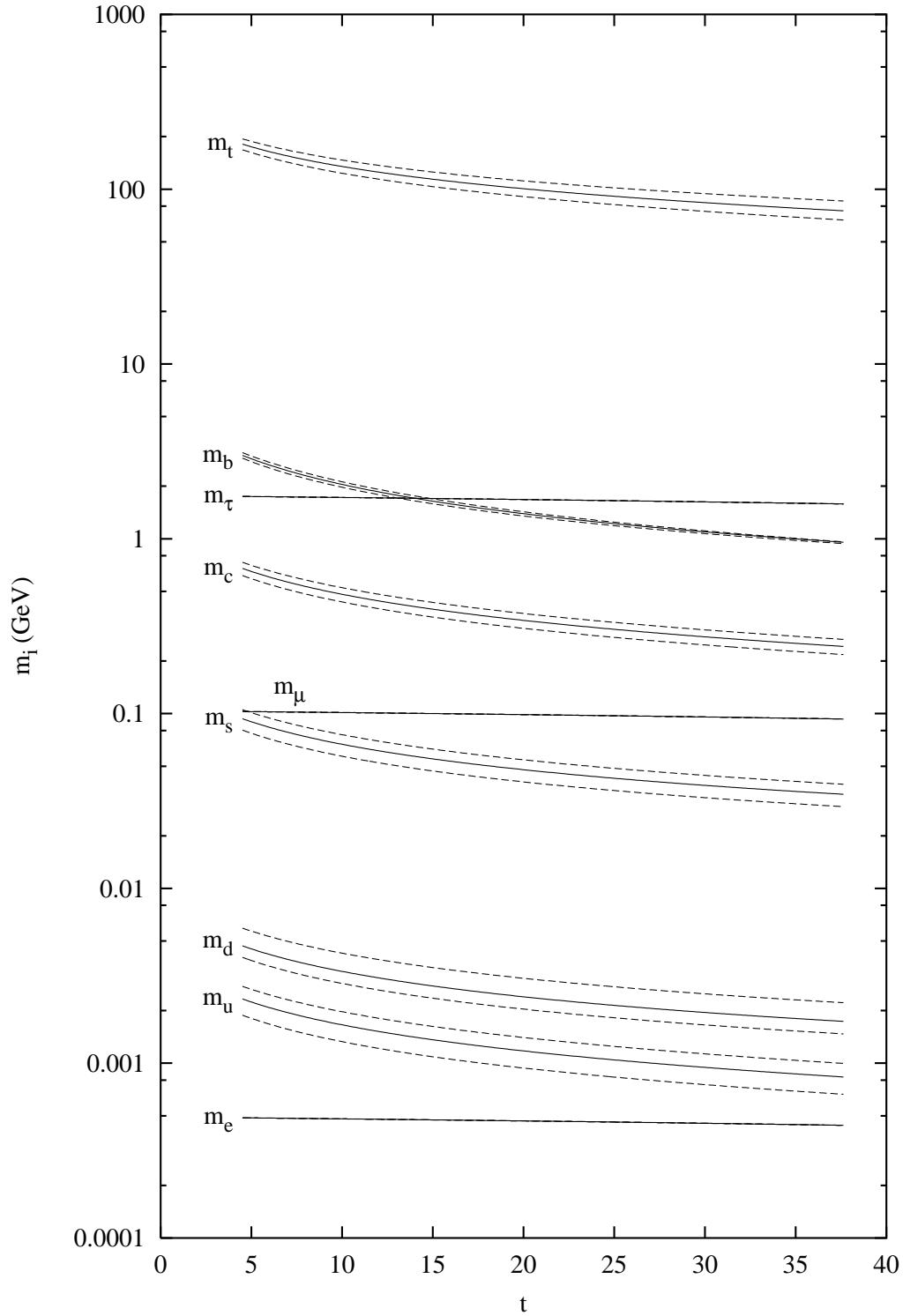


FIG. 6. Predictions of running masses at higher scales in SM with the input parameters given in (5.1)-(5.3) and  $M_H = 250$  GeV.

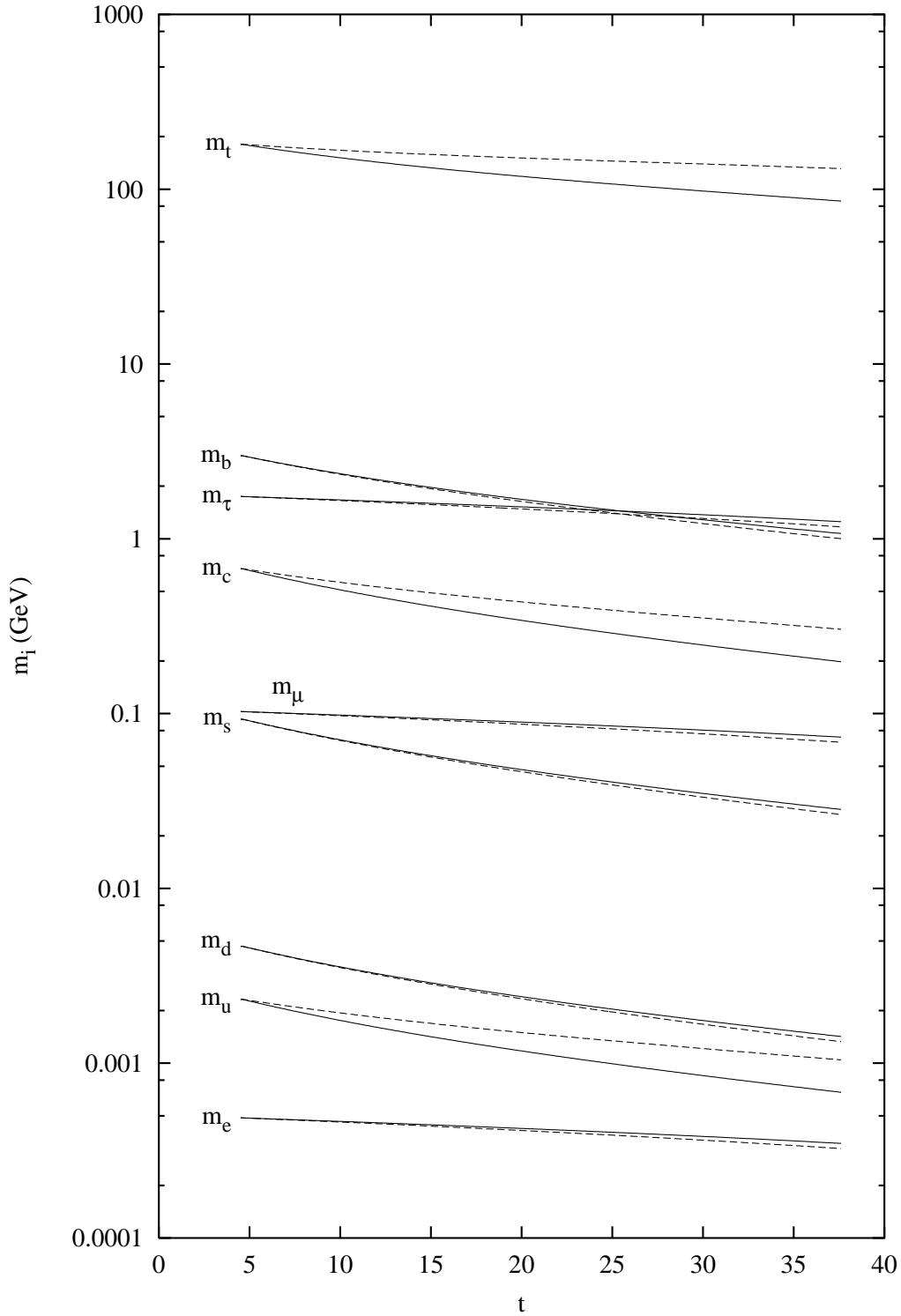


FIG. 7. Comparison of running mass predictions in the MSSM (solid lines) with those obtained from scale-independent assumptions (dashed lines) on the VEVs. The SUSY scale has been taken to be  $M_Z$ .

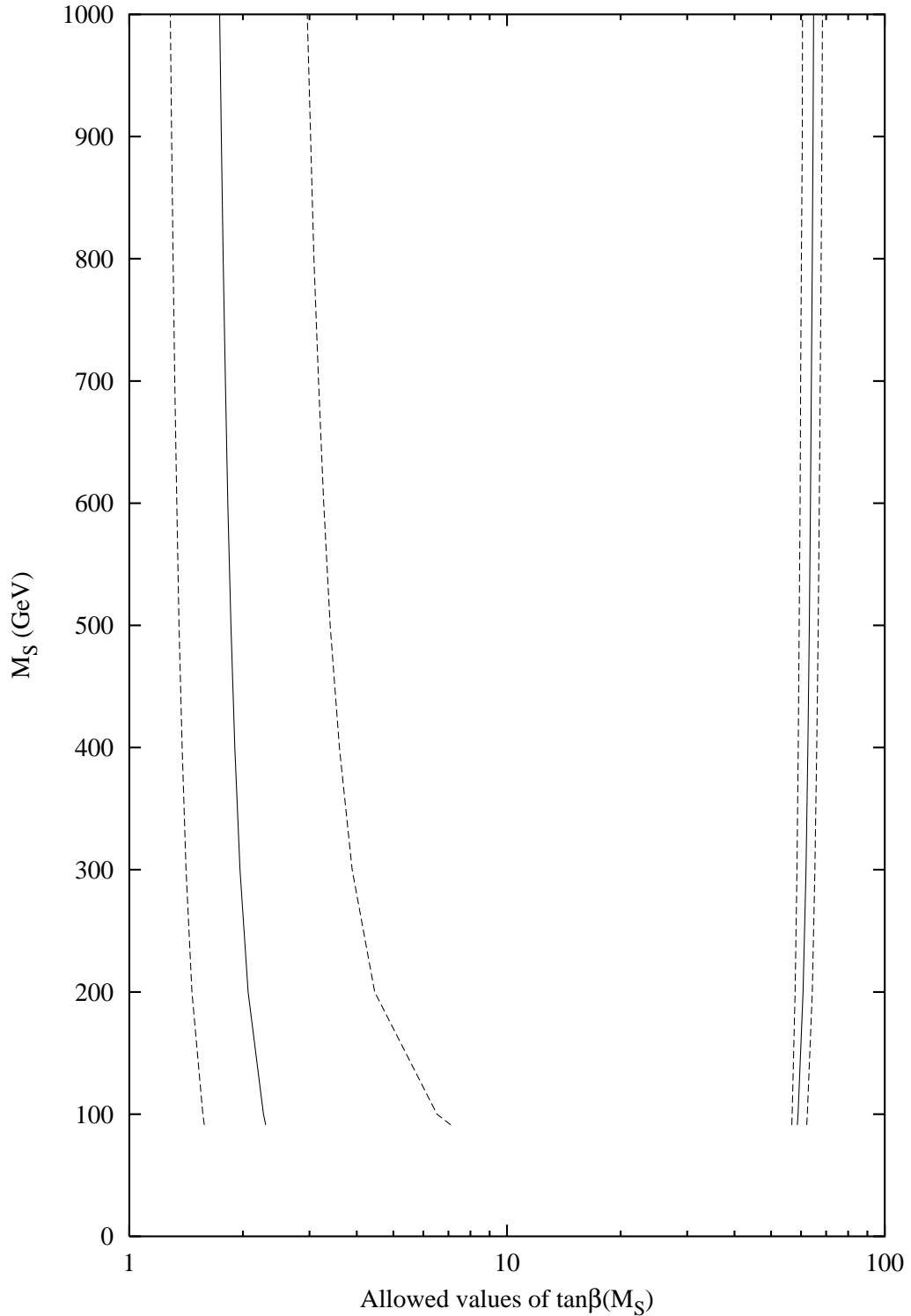


FIG. 8. Perturbatively allowed region for  $\tan\beta(M_S)$  as a function of SUSY scale  $M_S$ . The lower (upper) limits are due to top-quark ( $b$ -quark) Yukawa coupling. The dashed lines are due to uncertainties in the respective input masses.

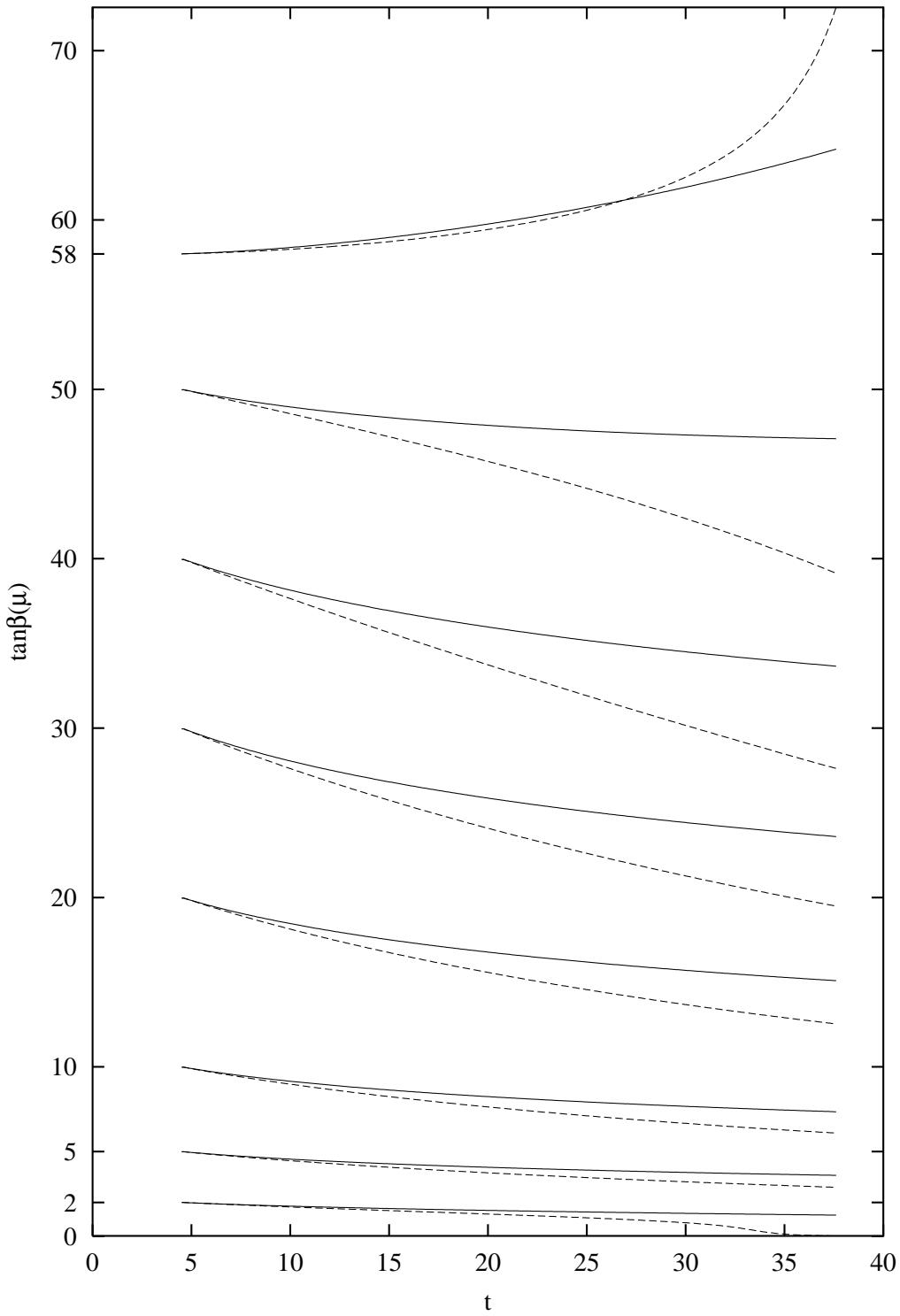


FIG. 9. Variation of  $\tan \beta(\mu)$  as a function of  $\mu$  ( $t = \ln \mu$ ) for different input values of  $\tan \beta(M_Z)$  in MSSM (solid lines) and 2HDM (dashed lines). The inputs for different curves starting from the bottom most line is  $\tan \beta(M_Z) = 2, 5, 10, 20, 30, 40, 50$  and  $58$ . In the MSSM the SUSY scale has been taken as  $M_Z$

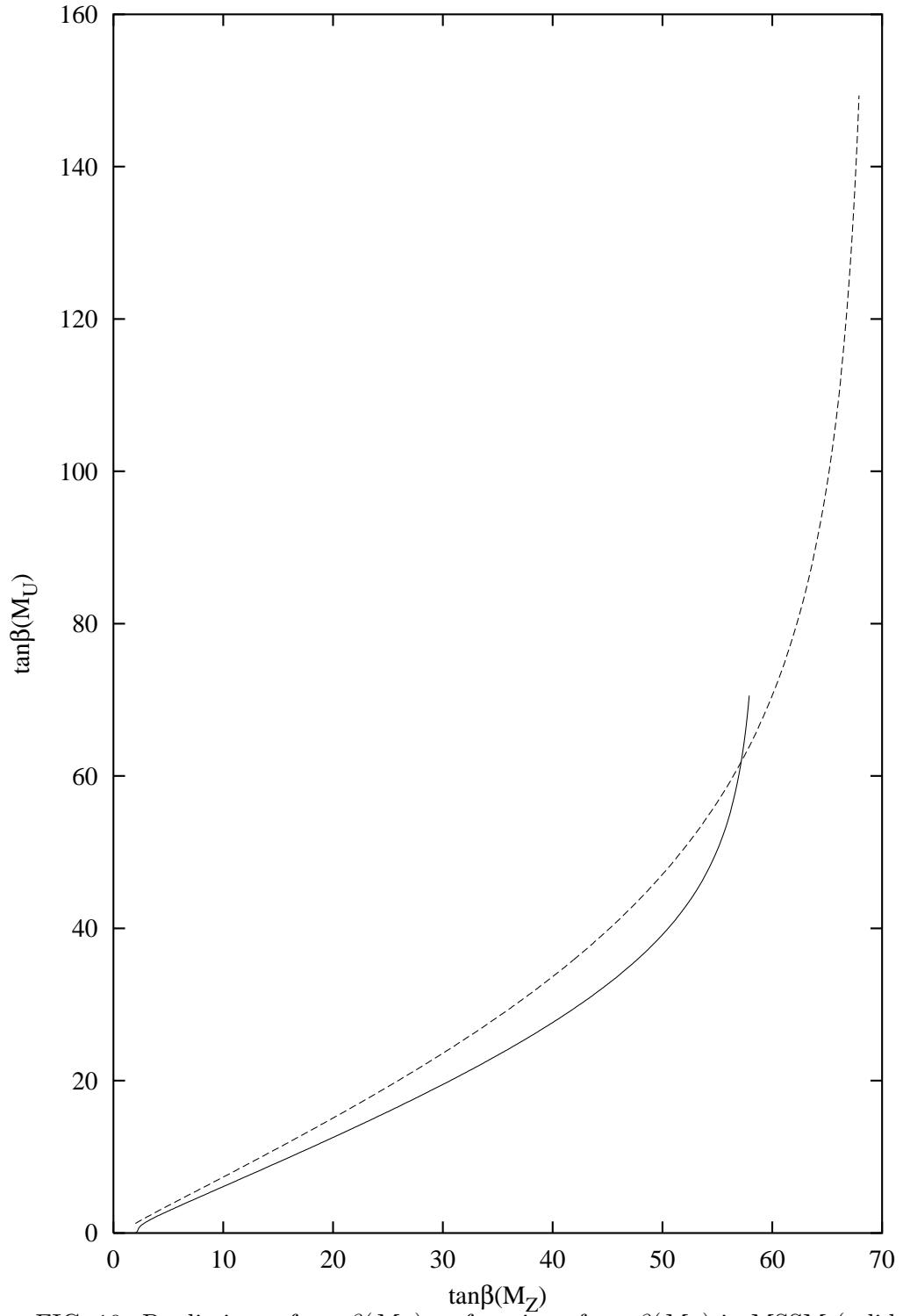


FIG. 10. Predictions of  $\tan \beta(M_U)$  as function of  $\tan \beta(M_Z)$  in MSSM (solid line) and 2HDM (dashed line). In MSSM the SUSY scale has been taken as  $M_Z$ .

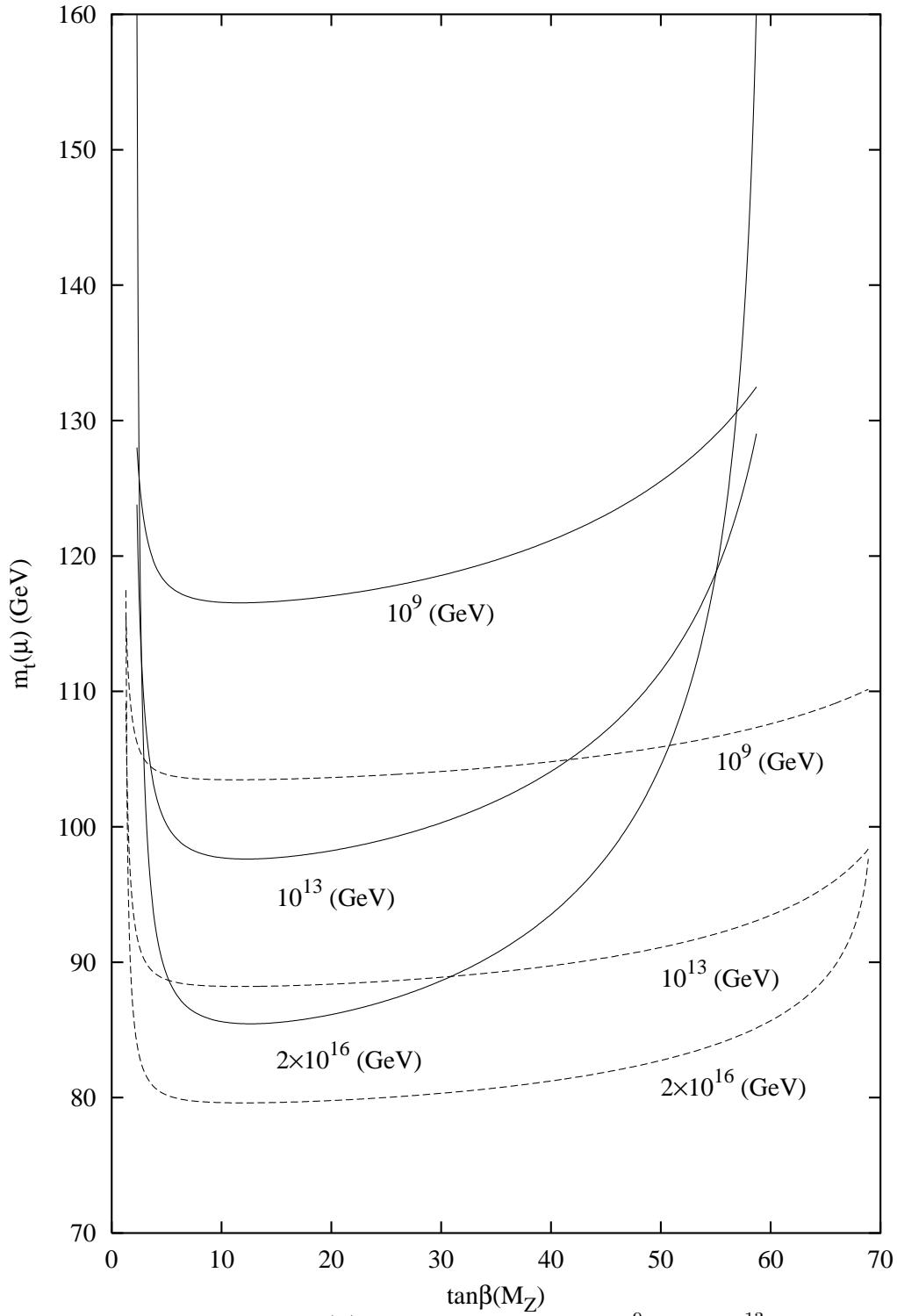


FIG. 11. Prediction of  $m_t(\mu)$  at higher scales,  $\mu = 10^9$  GeV,  $10^{13}$  GeV and  $2 \times 10^{16}$  GeV as a function of  $\tan \beta(M_Z)$  in MSSM with  $M_S = M_Z$  (solid lines) and 2HDM (dashed lines).

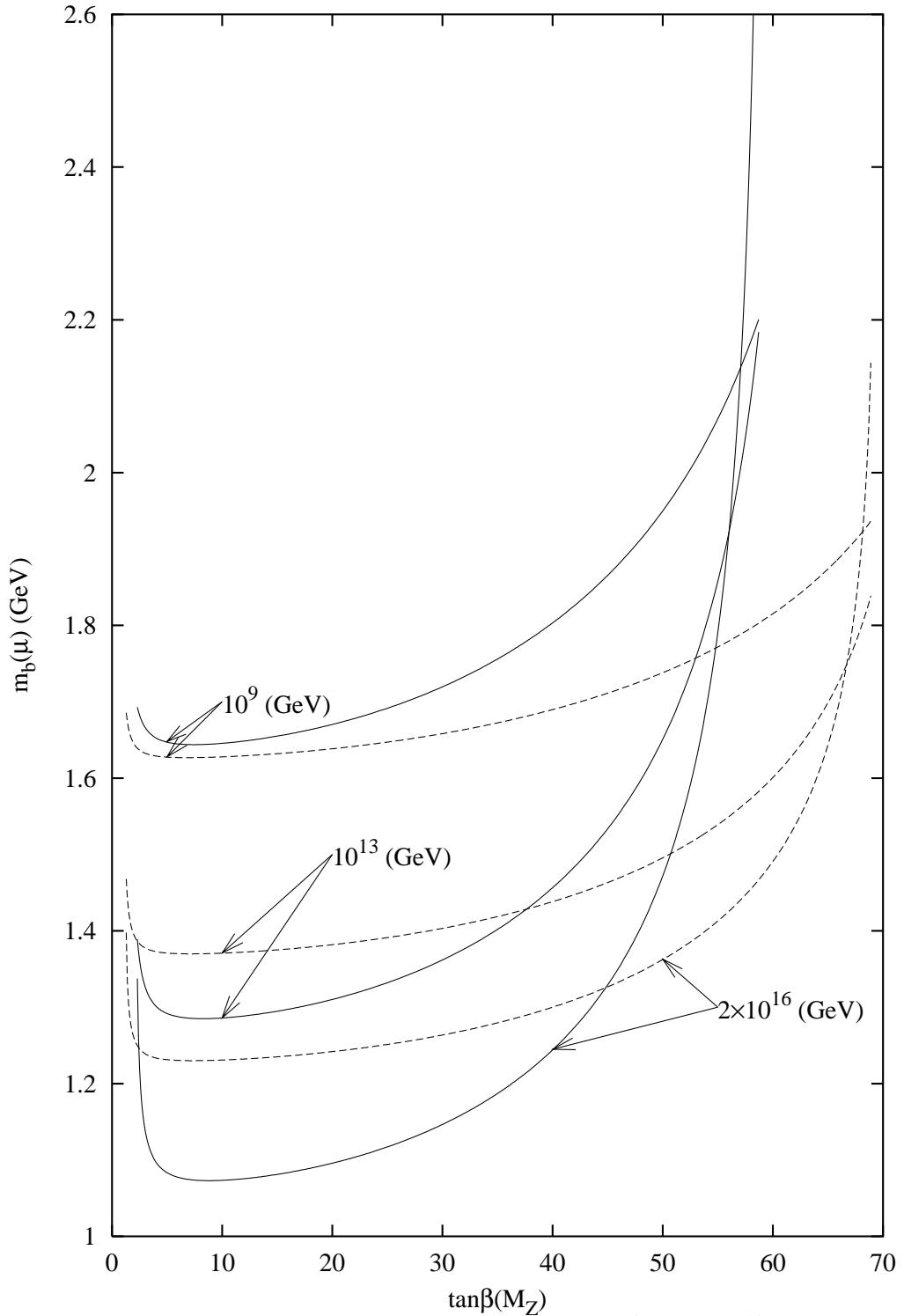


FIG. 12. Prediction of  $m_b(\mu)$  at higher scales,  $\mu = 10^9$  GeV,  $10^{13}$  GeV and  $2 \times 10^{16}$  GeV as a function of  $\tan \beta(M_Z)$  in MSSM with  $M_S = M_Z$  (solid lines) and 2HDM (dashed lines).

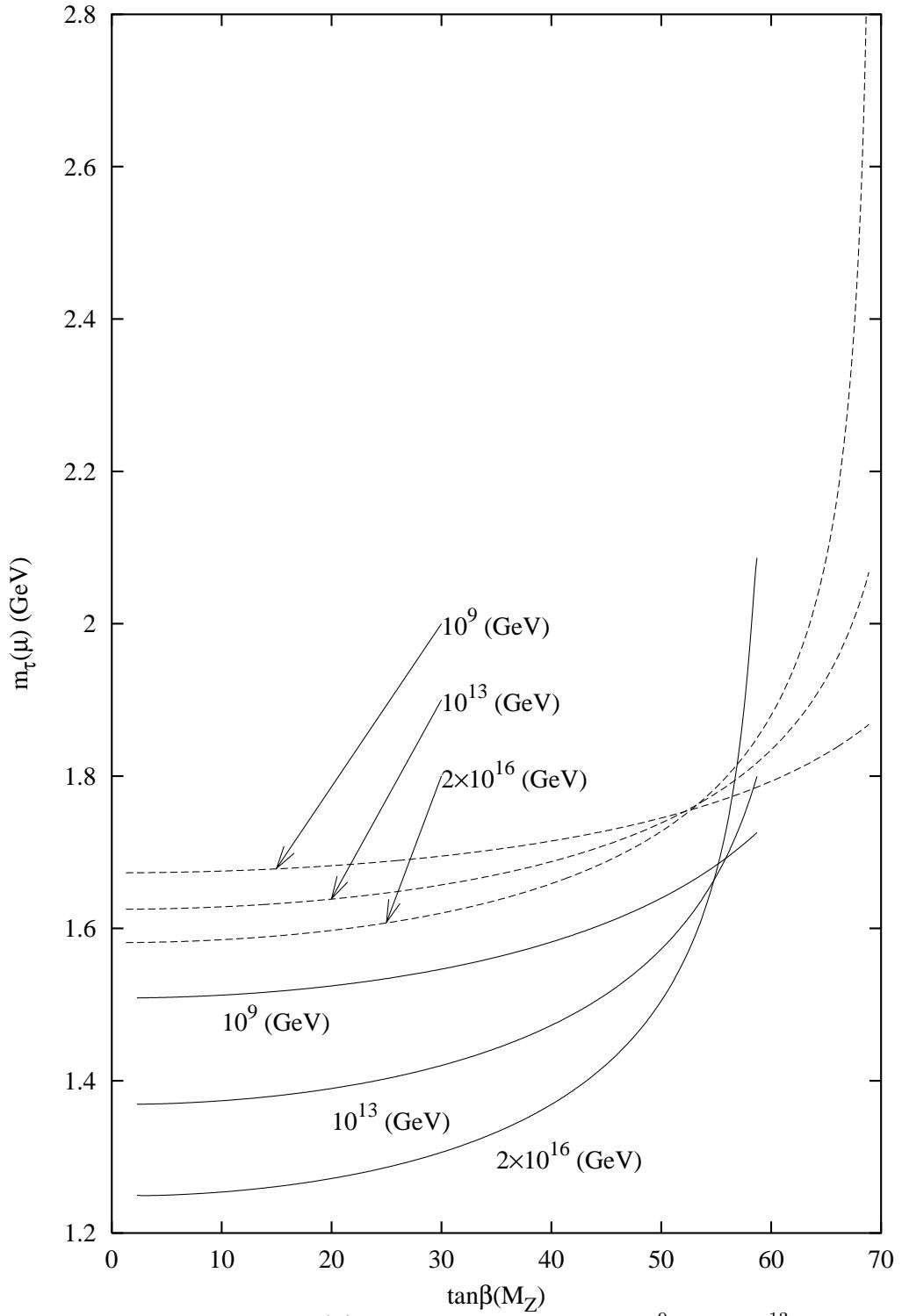


FIG. 13. Prediction of  $m_\tau(\mu)$  as at higher scales,  $\mu = 10^9$  GeV,  $10^{13}$  GeV and  $2 \times 10^{16}$  GeV as a function of  $\tan \beta(M_Z)$  in MSSM with  $M_S = M_Z$  (solid lines) and 2HDM (dashed lines).

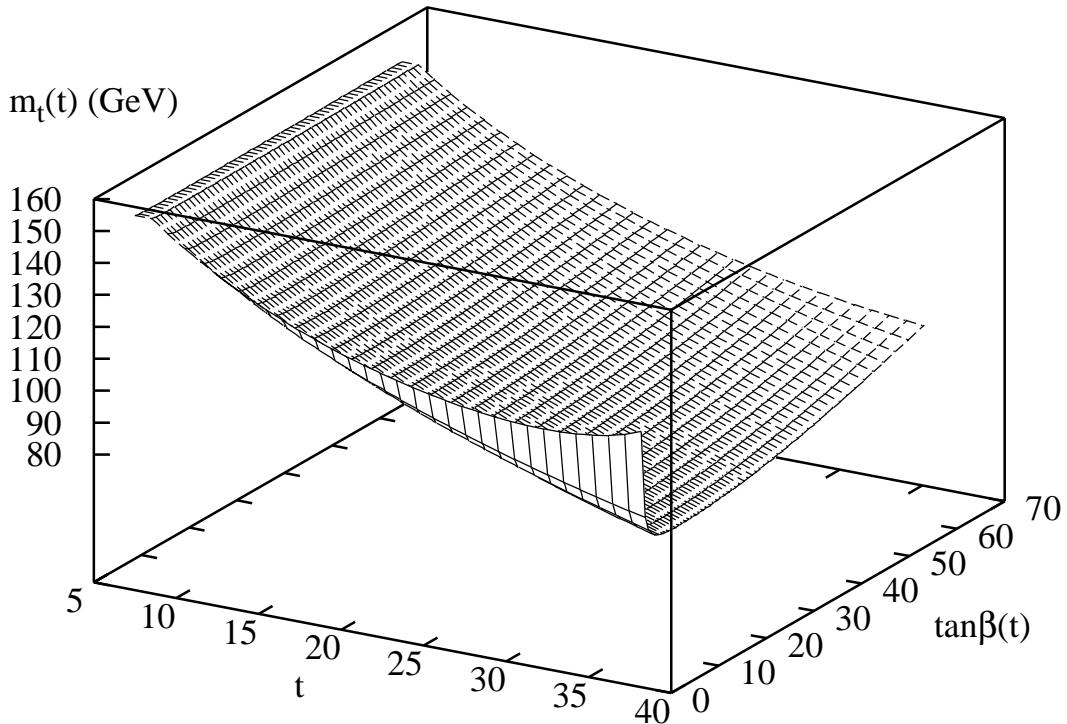


FIG. 14. Prediction of top-quark mass  $m_t(\mu)$  at higher scales ( $\mu > M_Z$ ) as a function of  $\mu$  ( $t = \ln \mu$ ) and  $\tan \beta(\mu)$  in MSSM with  $M_S = 1$  TeV. The values of  $\tan \beta(\mu)$  at very  $\mu$  has been obtained through solutions of the corresponding RGE using  $\tan \beta(M_Z) = 2 - 58$  as inputs.

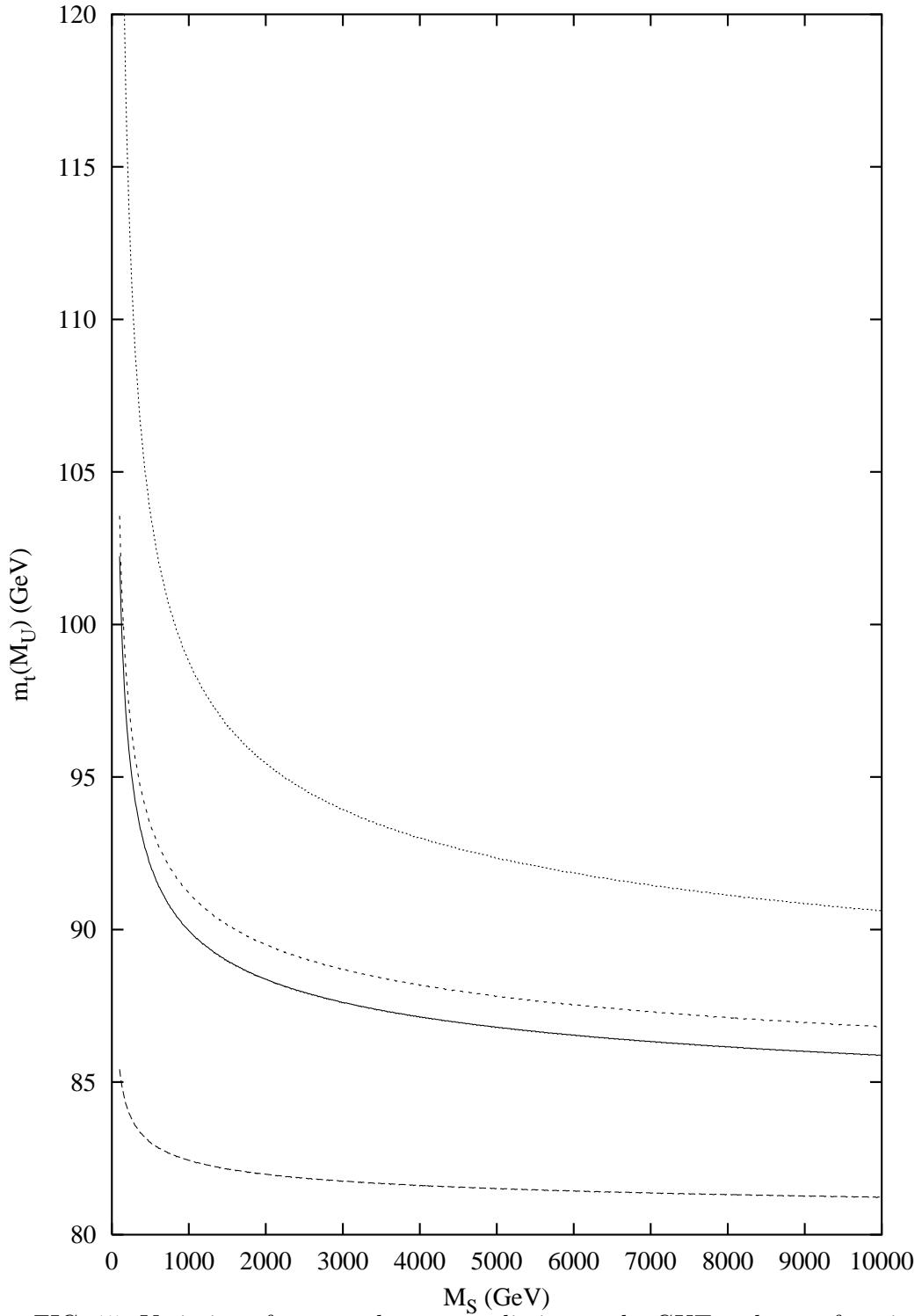


FIG. 15. Variation of top-quark mass prediction at the GUT scale as a function of SUSY scale

$M_S = M_Z - 10^4$  GeV and various values of  $\tan \beta(M_S) = 3$  (solid line), 10 (large-dashed line), 50 (small-dashed line), 55 (dotted line).

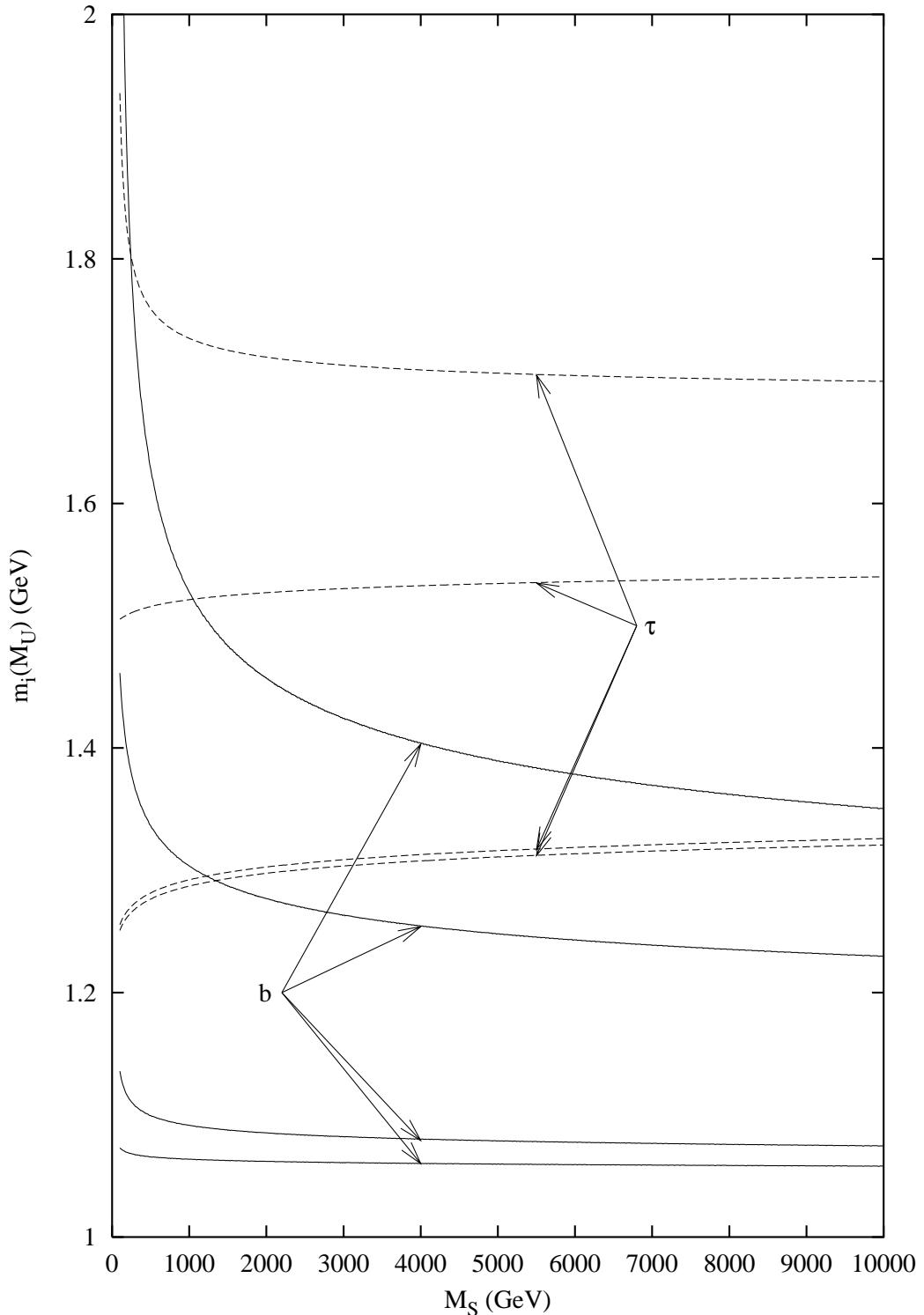


FIG. 16. Same as Fig. 15 but for  $b$ -quark and  $\tau$ -lepton mass predictions. The input values of  $\tan \beta(M_S)$  for four different curves in each case are  $\tan \beta(M_S) = 3, 10, 50$ , and  $55$  in the increasing order of masses.

## TABLES

TABLE I. Running mass and VEV predictions at higher scales in the nonSUSY standard model for the input values of the Higgs mass  $M_H = 250$  GeV and other parameters given in (5.1)-(5.3).

	$\mu = 10^9$ (GeV)	$\mu = 10^{13}$ (GeV)	$\mu = 2 \times 10^{16}$ (GeV)
$m_u$ (MeV)	$1.1537^{+0.2233}_{-0.2331}$	$0.9472^{+0.1849}_{-0.1923}$	$0.8351^{+0.1636}_{-0.1700}$
$m_c$ (MeV)	$335.2184^{+31.8261}_{-33.5603}$	$275.2419^{+26.5286}_{-27.8710}$	$242.6476^{+23.5536}_{-24.7026}$
$m_t$ (GeV)	$99.1359^{+10.7438}_{-9.8347}$	$83.9249^{+10.2622}_{-9.0281}$	$75.4348^{+9.9647}_{-8.5401}$
$m_d$ (MeV)	$2.3558^{+0.6513}_{-0.3538}$	$1.9529^{+0.5433}_{-0.2953}$	$1.7372^{+0.4846}_{-0.2636}$
$m_s$ (MeV)	$46.9155^{+6.5228}_{-6.9737}$	$38.8929^{+5.4652}_{-5.8228}$	$34.5971^{+4.8857}_{-5.1971}$
$m_b$ (GeV)	$1.3639^{+0.0328}_{-0.0398}$	$1.0971^{+0.0143}_{-0.0248}$	$0.9574^{+0.0037}_{-0.0169}$
$m_e$ (MeV)	$0.4665^{+0.0001}_{-0.0001}$	$0.4533^{+0.0001}_{-0.0001}$	$0.4413^{+0.0001}_{-0.0001}$
$m_\mu$ (MeV)	$98.4648^{+0.0049}_{-0.0050}$	$95.6834^{+0.0078}_{-0.0084}$	$93.1431^{+0.0136}_{-0.0101}$
$m_\tau$ (GeV)	$1.6738^{+0.0004}_{-0.0003}$	$1.6265^{+0.0005}_{-0.0004}$	$1.5834^{+0.0001}_{-0.0005}$
$v$ (GeV)	$157.5206^{+7.1815}_{-6.0558}$	$155.7062^{+10.6592}_{-8.5945}$	$155.6196^{+13.6336}_{-10.4664}$

TABLE II. Predictions of running masses, VEVs and  $\tan \beta$  at higher scales  $\mu = 10^9$  GeV,  $10^{13}$  GeV and  $2 \times 10^{16}$  GeV in MSSM with SUSY scale  $M_S = 1$  TeV, using two-loop RG equations.

$\tan \beta(M_S) = 10$	$\mu = 10^9$ (GeV)	$\mu = 10^{13}$ (GeV)	$\mu = 2 \times 10^{16}$ (GeV)
$m_u$ (MeV)	$1.1618^{+0.2226}_{-0.2345}$	$0.8882^{+0.1694}_{-0.1794}$	$0.7238^{+0.1365}_{-0.1467}$
$m_c$ (MeV)	$339.4064^{+31.2929}_{-33.4804}$	$258.0945^{+23.8287}_{-25.8339}$	$210.3273^{+19.0036}_{-21.2264}$
$m_t$ (GeV)	$112.3144^{+17.0392}_{-13.7215}$	$94.3698^{+22.5577}_{-14.4831}$	$82.4333^{+30.2676}_{-14.7686}$
$m_d$ (MeV)	$2.3842^{+0.6582}_{-0.3574}$	$1.8290^{+0.5111}_{-0.2779}$	$1.5036^{+0.4235}_{-0.2304}$
$m_s$ (MeV)	$47.4812^{+6.5845}_{-7.0454}$	$36.4261^{+5.1588}_{-5.4807}$	$29.9454^{+4.3001}_{-4.5444}$
$m_b$ (GeV)	$1.5920^{+0.1038}_{-0.0915}$	$1.2637^{+0.1189}_{-0.0893}$	$1.0636^{+0.1414}_{-0.0865}$
$m_e$ (MeV)	$0.4290^{+0.0001}_{-0.0001}$	$0.3911^{+0.0002}_{-0.0002}$	$0.3585^{+0.0003}_{-0.0003}$
$m_\mu$ (MeV)	$90.5439^{+0.0169}_{-0.0173}$	$82.5539^{+0.0346}_{-0.0330}$	$75.6715^{+0.0578}_{-0.0501}$
$m_\tau$ (GeV)	$1.5429^{+0.0006}_{-0.0006}$	$1.4085^{+0.0009}_{-0.0008}$	$1.2922^{+0.0013}_{-0.0012}$
$\tan \beta$	$8.2314^{+0.5046}_{-0.3807}$	$7.4350^{+0.9752}_{-0.6302}$	$6.9280^{+1.5156}_{-0.8234}$
$v_u$ (GeV)	$141.7765^{+9.7365}_{-7.6253}$	$130.5455^{+18.0431}_{-12.1155}$	$123.8177^{+27.8954}_{-15.7651}$
$v_d$ (GeV)	$17.2237^{+0.1352}_{-0.1241}$	$17.5581^{+0.1426}_{-0.1302}$	$17.8718^{+0.1492}_{-0.1354}$

$\tan \beta(M_S) = 55$	$\mu = 10^9$ (GeV)	$\mu = 10^{13}$ (GeV)	$\mu = 2 \times 10^{16}$ (GeV)
$m_u$ (MeV)	$1.1687^{+0.2225}_{-0.2346}$	$0.8889^{+0.1675}_{-0.1795}$	$0.7244^{+0.1219}_{-0.1466}$
$m_c$ (MeV)	$339.5917^{+31.2621}_{-33.5026}$	$258.2929^{+23.3295}_{-25.8144}$	$210.5049^{+15.1077}_{-21.1538}$
$m_t$ (GeV)	$118.6588^{+19.9035}_{-15.4790}$	$104.2363^{+32.7015}_{-18.2028}$	$95.1486^{+69.2836}_{-20.659}$
$m_d$ (MeV)	$2.3774^{+0.6542}_{-0.3553}$	$1.8219^{+0.5054}_{-0.2755}$	$1.4967^{+0.4157}_{-0.2278}$
$m_s$ (MeV)	$47.3523^{+6.5303}_{-7.0069}$	$36.2891^{+5.0777}_{-5.4340}$	$29.8135^{+4.1795}_{-4.4967}$
$m_b$ (GeV)	$1.8297^{+0.1667}_{-0.1376}$	$1.5768^{+0.2640}_{-0.1685}$	$1.4167^{+0.4803}_{-0.1944}$
$m_e$ (MeV)	$0.4276^{+0.0003}_{-0.0001}$	$0.3893^{+0.0005}_{-0.0002}$	$0.3565^{+0.001}_{-0.0002}$
$m_\mu$ (MeV)	$90.2779^{+0.0508}_{-0.0318}$	$82.2064^{+0.1024}_{-0.0468}$	$75.2938^{+0.1912}_{-0.0515}$
$m_\tau$ (GeV)	$1.6867^{+0.0056}_{-0.005}$	$1.6574^{+0.0188}_{-0.0148}$	$1.6292^{+0.0443}_{-0.0294}$
$\tan \beta$	$53.6122^{+2.3644}_{-1.5356}$	$52.7633^{+6.3597}_{-2.9538}$	$52.0738^{+16.5475}_{-4.3757}$
$v_u$ (GeV)	$141.2095^{+10.6285}_{-8.1355}$	$127.4742^{+22.6973}_{-13.8538}$	$117.7947^{+46.7214}_{-19.2752}$
$v_d$ (GeV)	$2.6339^{+0.0859}_{-0.0741}$	$2.4159^{+0.158}_{-0.1206}$	$2.2620^{+0.2615}_{-0.1661}$

TABLE III. Predictions of running masses, VEVs and  $\tan \beta$  in 2HDM at higher scales using one-loop RG equations.

$\tan \beta(M_S) = 10$	$\mu = 10^9$ (GeV)	$\mu = 10^{13}$ (GeV)	$\mu = 2 \times 10^{16}$ (GeV)
$m_u$ (MeV)	$1.2021^{+0.2309}_{-0.2417}$	$0.9908^{+0.1919}_{-0.2002}$	$0.8749^{+0.1701}_{-0.1772}$
$m_c$ (MeV)	$349.2805^{+32.6824}_{-34.5798}$	$287.8975^{+27.3606}_{-28.8305}$	$254.2131^{+24.3398}_{-25.5998}$
$m_t$ (GeV)	$103.5011^{+11.3400}_{-10.2307}$	$88.2332^{+11.1753}_{-9.5397}$	$79.6373^{+11.1974}_{-9.127}$
$m_d$ (MeV)	$2.4547^{+0.6748}_{-0.366}$	$2.0430^{+0.5650}_{-0.3069}$	$1.8204^{+0.505}_{-0.2743}$
$m_s$ (MeV)	$48.8852^{+6.7278}_{-7.2144}$	$40.6860^{+5.6602}_{-6.0484}$	$36.2544^{+5.0700}_{-5.4083}$
$m_b$ (GeV)	$1.6281^{+0.0910}_{-0.0854}$	$1.3709^{+0.0854}_{-0.0775}$	$1.2309^{+0.0826}_{-0.0730}$
$m_e$ (MeV)	$0.4662^{+0.0001}_{-0.0001}$	$0.4529^{+0.0001}_{-0.0001}$	$0.4407^{+0.0001}_{-0.0001}$
$m_\mu$ (MeV)	$98.4132^{+0.0050}_{-0.0051}$	$95.5970^{+0.0086}_{-0.0086}$	$93.0197^{+0.0122}_{-0.0122}$
$m_\tau$ (GeV)	$1.6752^{+0.0004}_{-0.0004}$	$1.6283^{+0.0004}_{-0.0004}$	$1.5851^{+0.0005}_{-0.0005}$
$\tan \beta$	$8.1956^{+0.3894}_{-0.3255}$	$7.6757^{+0.5649}_{-0.4496}$	$7.3543^{+0.6975}_{-0.5348}$
$v_u$ (GeV)	$155.6481^{+7.4729}_{-6.2622}$	$152.8315^{+11.3442}_{-9.0595}$	$151.9551^{+14.5219}_{-11.1741}$
$v_d$ (GeV)	$18.9914^{+0.0098}_{-0.0095}$	$19.9110^{+0.0137}_{-0.0131}$	$20.6620^{+0.0167}_{-0.0157}$

$\tan \beta(M_S) = 55$	$\mu = 10^9$ (GeV)	$\mu = 10^{13}$ (GeV)	$\mu = 2 \times 10^{16}$ (GeV)
$m_u$ (MeV)	$1.2021^{+0.2309}_{-0.2417}$	$0.9908^{+0.1919}_{-0.2002}$	$0.8749^{+0.1701}_{-0.1772}$
$m_c$ (MeV)	$349.2889^{+32.6905}_{-34.5782}$	$287.9066^{+27.3646}_{-28.8338}$	$254.2223^{+24.3441}_{-25.6031}$
$m_t$ (GeV)	$106.6700^{+12.2719}_{-10.9231}$	$92.1000^{+12.6648}_{-10.5174}$	$83.9317^{+13.2279}_{-10.3226}$
$m_d$ (MeV)	$2.4547^{+0.6748}_{-0.3660}$	$2.0430^{+0.5650}_{-0.3069}$	$1.8204^{+0.5050}_{-0.2743}$
$m_s$ (MeV)	$48.8888^{+6.7295}_{-7.2159}$	$40.6898^{+5.6622}_{-6.0498}$	$36.2584^{+5.0720}_{-5.4099}$
$m_b$ (GeV)	$1.7719^{+0.1203}_{-0.1092}$	$1.5392^{+0.1272}_{-0.1092}$	$1.4128^{+0.1353}_{-0.1162}$
$m_e$ (MeV)	$0.4662^{+0.0001}_{-0.0001}$	$0.4529^{+0.0001}_{-0.0001}$	$0.4407^{+0.0001}_{-0.0001}$
$m_\mu$ (MeV)	$98.4302^{+0.0054}_{-0.0055}$	$95.6235^{+0.0097}_{-0.0094}$	$93.0536^{+0.0146}_{-0.0136}$
$m_\tau$ (GeV)	$1.7659^{+0.0028}_{-0.0025}$	$1.7775^{+0.0073}_{-0.0060}$	$1.7851^{+0.0136}_{-0.0107}$
$\tan \beta$	$54.9963^{+1.5534}_{-1.1589}$	$55.7094^{+2.4787}_{-1.6588}$	$56.5831^{+3.2730}_{-1.9895}$
$v_u$ (GeV)	$155.7178^{+7.8644}_{-6.5265}$	$152.0846^{+12.3141}_{-9.6439}$	$150.4478^{+16.1914}_{-12.0879}$
$v_d$ (GeV)	$2.8314^{+0.0649}_{-0.0578}$	$2.7299^{+0.1042}_{-0.0892}$	$2.6588^{+0.1404}_{-0.1161}$